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The shape of nonabelian D-branes

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Abstract: We evaluate bulk distribution of energies, pressures and various D-brane/F-string charges generated by nontrivial matrix configurations in nonabelian D-brane effective field theories, using supergravity source density formulas derived originally in Matrix theory. Off-diagonal elements of worldvolume nonabelian fields, especially transverse scalar fields, induce various interesting bulk structures exhibiting the shape of branes. First, we study the energy distribution of string-brane networks generated in the bulk by the Yang-Mills monopoles and the 1/4 BPS dyons, and confirm force balance of them. An application to the Yang-Mills description of recombination of intersecting D-branes gives results indicating presence of the tachyon matter. Second, we analyse the shape of fuzzy D-branes given by nonabelian scalar fields which are mutually noncommutative. We employ fuzzy S^2, fuzzy S^4 and fuzzy cylinder/supertube as matrix configurations of N D0-branes representing higher dimensional noncommutative D-branes. We find that in the continuum (large-N) limit the D-brane charge distributions become in the expected shape of a sphere or a cylinder with an infinitesimal thickness. However, the distributions found for finite N are difficult to interpret, which leaves a puzzle for a possible dual description in terms of higher dimensional D-branes. A resolution is provided with use of an ordering ambiguity in the charge density formulas.

Keywords: D-branes, Non-Commutative Geometry.

Contents

1. Introduction and summary

The idea of *brane democracy*, originally used by P. K. Townsend [1] in the context of superalgebras of low energy supergravity in string theories, has an interesting possibility to be a guiding principle to seek for a unified description of string theories. Looking back the developments in the last decade, as candidates for the unified description, one may find Matrix theory and IIB matrix model which adopt lower dimensional D-branes as fundamental constituents, as well as tachyon condensation scenario of K-theory in which one might start with the highest dimensional unstable D-branes on the other hand. It is unclear why these descriptions based on D-branes with different dimensions can be consistent, but we might have relied substantially on the general idea of the brane democracy — all the physics in string/M-theory can be constructed by starting with any dimensional D-branes. The interplay between the above different descriptions is still rather obscure, leaving the question that what kind of objects in string theories can be consistently described in different scenarios.

One of the simplest examples in which a single brane object can be described in two equivalent ways is a flat D2-D0 bound state. It can be described by (i) a flat D2-brane with a constant magnetic field on it, and also by (ii) infinite number of D0-branes whose representation matrices satisfy Heisenberg algebra. These two descriptions give the same boundary state [2] and thus describe completely identical physics. The essential part of the equivalence is that, although the second description uses the D0-branes as constituents, their matrix-valued configurations provide "fuzzy" plane whose support is just a flat R^2 which is identical with the geometry of the first description.

Next natural question which one might ask is to what extent this "democratic" duality on the flat D2-D0 bound state can be generalized. To answer this challenging question partly, in this paper we investigate geometric shapes produced by various fuzzy ($\sim$ matrix-valued, or noncommutative) configurations of D0-branes (corresponding to (ii) in the above example), to see if they are consistent with the expected dual descriptions in terms of higher dimensional D-branes ((i) in the above). To extract the geometry of the fuzzy objects, we look at low energy sector of closed string excitations whose sources are generated by the matrix configurations of D0-branes. We make full use of the supergravity coupling to the low energy D0-brane effective field theory derived in [3].

Even for a finite number of D0-branes, one can argue the existence of this "democratic" duality in the following way. Let us consider N D0-branes placed in a background constant RR (Ramond-Ramond) 4-form field strength. As shown in [5], these form a fuzzy S^2 which is a classical solution of the D0-brane effective field theory in this background. On the other hand, one finds a spherical D2-brane on which N units of a magnetic field are turned on, as a classical solution of a D2-brane effective field theory in this background. This has the same number of the net D0-brane charge as the above fuzzy S^2 solution. Hence one can expect that these two might be the same object.

The aim of this paper is to make full use of the supergravity charge density formulas to study the geometrical information hidden in nonabelian configurations of D-brane worldvolume theories, and furthermore to see if this sort of expectation for the "democratic" duality with finite N is realized or not. We shall give a summary of the results of this paper in the following.

In section 2 we review the supergravity charge density formulas especially for D0-brane actions [3] and see how the noncommutative (fuzzy) plane can give the consistent democratic duality in them. The essence of the use of the supergravity charge density formulas [3] is that one can extract physical and gauge-invariant quantities from Matrix-valued, or equivalently, non-commutative or fuzzy, configurations of nonabelian D-brane worldvolume theories. Previously, the formulas were utilized to show the nontrivial brane structures such as the presence of strings extended away from D-brane worldvolumes, when nonabelian Yang-Mills configurations including BPS monopoles are employed [4]. In section 3, in order to show the power of the supergravity charge density formulas, we compute bulk distribution of energy-momentum tensor for the worldvolume solutions considered in [6] and [4]. We start with a computation of the energy and the pressure generated in the bulk by the solutions provided in [6] (a Yang-Mills description of recombination of intersecting D-branes). This explicitly shows that, after the recombination, there remains energy between the recombined D-branes which was indicated in [7]. The pressure of this unknown "matter" tends to vanish as time develops, hence we may regard this as an appearance of a tachyon matter [8] in Yang-Mills theory. We computed also the energy and pressure distributions of the supersymmetric solutions considered in [4]. It is shown that the presence of the D/F-strings away from the D-brane worldvolumes is necessary for the force balance of the whole configurations. For every part of the distributed strings, their charges and energies are consistent with the BPS bounds in the bulk with expected supersymmetries. Furthermore, we calculate string charges in the bulk generated in 1/4 BPS dyons in 1+3 dimensional Yang-Mills theory [9, 10, 11]. The curved shape of the D3-branes observed in [9, 10] is found to have an interpretation consistent with bulk (p, q)-strings present away from the D3-brane worldvolume.

A conclusion in section 3 is that, with a single transverse scalar field (or mutually commutative scalar fields), the supergravity charge density formulas give consistent brane configurations in the bulk. In the subsequent sections, we come to the main point of this paper: the study of the shape of fuzzy D-branes defined by mutually noncommutative scalar fields. We apply the RR density formulas to a fuzzy S^2, a fuzzy S^4, and a fuzzy cylinder, in section 4, 5 and 6 respectively. In section 6, we study also a fuzzy supertube as an application. We see that in the continuum limit ($\sim$ a large N limit) their RR charge distributions become in the shape of a sphere (or a cylinder) with an infinitesimal thickness, as expected. However, we find that for finite N, the charge distributions are different from what we expect. For the finite N, the charges are on a collection of shells with various radii, not on a single shell in the expected shape. Interestingly, if we take the large-N (continuum) limit, these shells annihilate with each other and only a single outer shell remains, to give results consistent with the democratic duality.

The last section is devoted to discussions for possible interpretation of these strange distributions for the finite N, as well as on the existence of the democratic duality at finite N. Assuming that the symmetrized ordering for the charge density formulas may receive corrections of the form of commutators, we compute the first nontrivial corrections by demanding the democratic duality.

In the course of the study, it turns out that the supergravity charge density formulas for matrix configurations of nonabelian worldvolume theories of D-branes are surprisingly powerful to reveal the actual information of the brane configuration in the bulk. We see various examples in which the formulas give new and consistent brane configurations extended in the bulk space. For cases with a single scalar field (or with several scalars which are commuting with each other), the formulas give completely consistent pictures of bulk D-brane/F-string configurations. However, for the cases with two or more noncommutative scalar fields the symmetrized trace prescription for the formulas doesn't seem to give bulk distribution consistent with the democratic duality, and we may need some correction to the ordering in the definition of the formulas. Unfortunately on our first motivation for understanding of the precise democratic duality, the supergravity charge density formulas for the latter cases turn out to be not that helpful, but we find that the formulas capture various intrinsic features of nonabelian worldvolume physics of D-branes. A more precise formulation of the correspondence between nonabelian worldvolume fields and the bulk charge distribution will give us a hint for the democratic duality and more unified view of nonperturbative string theory.

2. Charge density formulas and fuzzy plane

2.1 Formulas

Any D-brane couples to supergravity fields as a source, since D-branes are defined as boundaries of worldsheets and thus are sources of closed strings. With a given D-brane configurations provided by vacuum expectation values of the massless excitations of open strings ending on the D-brane, in principle one can compute how the D-brane is coupled to the closed string excitations such as gravitons and RR gauge fields.

Here we review the supergravity charge density formulas derived in Matrix theory [3] (see also [12, 5, 13, 14]). Let us consider a matrix configuration X_i $(i = 1, 2, \ldots, 9)$ which is the vacuum expectation value of the scalar fields coming from the excitation of the string connecting the D0-branes. Then the D0-brane charge density in the momentum representation is given by

$$\widetilde{J}_0(\vec{k}) = \mathcal{T}_{\rm D0}\, \mathrm{Str}\left[e^{ik_iX_i}\right] = \mathcal{T}_{\rm D0}\, \mathrm{tr}\left[e^{ik_iX_i}\right]. \tag{2.1}$$

where k_i is a momentum in the target spacetime, and $\mathcal{T}_{\rm D0}$ is the tension of the D0-brane. Though the original definition uses symmetrized trace in which X_i is treated as a unit of symmetrization, the exponential is already in a symmetrized form, so we may use the normal trace to evaluate the density. After an inverse Fourier transformation we will obtain the actual D0-brane distribution in the bulk spacetime.

This formula can be interpreted also in terms of D-brane boundary states. Consider an overlap of a boundary state of a D0-brane located at $x_i = 0$ and a closed string state of $C_\mu^{\rm RR}$ (a RR 1-form gauge field), $\langle C_0^{\rm RR}(x_i)|B_{\rm D0}\rangle$. This should be proportional to $\delta(x_i)$ (whose momentum representation is a constant), since the boundary state is a source for closed string excitations. A generalization of the boundary state for distributed D0-branes should be written as

$$\operatorname{tr}\exp\left[i\oint d\sigma \hat{P}_i(\sigma)X_i\right]|B_{\rm D0}\rangle\,, \tag{2.2}$$

where X_i are the matrices defining the bulk distribution of the D0-branes, and $\hat{P}_i(\sigma)$ is a worldsheet current of spacetime translation evaluated on a worldsheet boundary. In evaluating the overlap with the RR state, only the zero mode k_i of $\hat{P}$ is effective, so essentially the overlap is proportional to (2.1) in the momentum representation.

We adopt the ordering of the matrices as specified by the symmetrized trace [12] in the formula (2.1) and in the following formulas, but there is no a priori reason why we have to choose this symmetrized ordering [15]. However, as seen in the paper [4] and as shown in the next section, for certain examples with a single transverse scalar field this ordering prescription seems to be completely consistent with the physical interpretation of the resultant brane charge distributions. On the other hand, when there are two or more nontrivial transverse scalars which are noncommutative with each other, with the above ordering prescription we encounter a strange result, as we will see in section 4, 5 and 6. Until section 7 where we consider a correction to the ordering, we shall use the symmetrized trace. The symmetrized ordering was confirmed also in some cases of string scattering amplitudes [14]. It is interesting to note that in the pp-wave/Super Yang-Mills correspondence [16] exactly the same kind of formulas appear with the symmetrized trace (see [17] for this aspect of the symmetrized trace). This resemblance is not accidental but has an origin in the derivation of BMN operators from disk amplitudes [18].

The formula for D2-brane charge density generated by the configuration X^i is

$$\widetilde{J}_{0ij}(\vec{k}) = \frac{\mathcal{T}_{\rm D0}}{2\pi\alpha'}\operatorname{Str}\left[-i[X_i,X_j]e^{ik_lX_l}\right]. \tag{2.3}$$

Here the symmetrization involves the commutator as another unit. It is easy to see that this symmetrized trace can be replaced by the normal trace, again. The D4-brane charge density is a bit more involved,

$$\widetilde{J}_{0ijkl}(\vec{k}) \equiv \frac{\mathcal{T}_{\rm D0}}{2(2\pi\alpha')^2}\epsilon_{ijklm}\epsilon_{mpqrs}\operatorname{Str}\left(X_pX_qX_rX_se^{ik_lX_l}\right). \tag{2.4}$$

In the parenthesis, $[X_i,X_j]$ should be regarded as a unit for the symmetrization, through (a trivial generalization of) a formula

$$\epsilon_{pqrs5}X_pX_qX_rX_s = \{[X_1,X_2],[X_3,X_4]\} + \{[X_2,X_3],[X_1,X_4]\} + \{[X_3,X_1],[X_2,X_4]\}\,,$$

where $p,q,r,s = 1,2,3,4$.

The coupling to the graviton gives the energy-momentum tensor generated in the bulk. The formulas are [3]

$$\widetilde{T}_{00}(\vec{k}) = \mathcal{T}_{\rm D0}\,{\rm Str}\left[\left(\mathbb{1} + \frac{1}{2}\dot{X}_i{}^2 - \frac{1}{4}\frac{1}{(2\pi\alpha')^2}[X_i,X_j]^2\right)e^{ik_lX_l}\right], \tag{2.5}$$

$$\begin{aligned}\widetilde{T}_{0i}(\vec{k}) &= \mathcal{T}_{\rm D0}\,{\rm Str}\left[\left(\dot{X}_i\left(\mathbb{1} + \frac{1}{2}\dot{X}_i{}^2 - \frac{1}{4(2\pi\alpha')^2}[X_i,X_j]^2\right) - \right.\right.\\ &\qquad\left.\left. - \frac{1}{(2\pi\alpha')^2}[X_i,X_j][X_j,X_k]\dot{X}_k\right)e^{ik_lX_l}\right],\\ \widetilde{T}_{ij}(\vec{k}) &= \mathcal{T}_{\rm D0}\,{\rm Str}\left[\left(\dot{X}_i\dot{X}_j - \frac{1}{(2\pi\alpha')^2}[X_i,X_k][X_k,X_j]\right)e^{ik_lX_l}\right].\end{aligned} \tag{2.6}$$

There should be higher order correction terms in this expression, especially a term like $[X,X]^4$ in T_{00}, since naively the zero momentum part ($k=0$) of the bulk energy-momentum tensor T_{00} may coincide with the hamiltonian computed from a D0-brane effective field theory, that is, a nonabelian Born-Infeld action plus its derivative corrections. The coupling to the NSNS (Neveu-Schwarz Neveu-Schwarz) B-field provides the bulk distribution of the F-string charge,

$$\widetilde{I}_{0i}(\vec{k}) = \frac{\mathcal{T}_{\rm D0}}{2\pi\alpha'}\,{\rm Str}\left[i[X_i,X_j]\dot{X}_je^{ik_lX_l}\right]. \tag{2.7}$$

In these formulas the unit for the symmetrization is $[X_i,X_j]$, $\dot{X}_j$ and k_lX_l.

2.2 Fuzzy plane

Let us illustrate how the RR charge density formulas are treated in the simplest example of a fuzzy plane (noncommutative plane). The fuzzy plane is defined by a nontrivial commutator

$$[X_1,X_2] = i\theta\,. \tag{2.8}$$

The representation of this algebra is infinite dimensional. Let us see how this algebra generates a higher dimensional charge. With use of an identity

$${\rm tr}\left[e^{ik\cdot X}\right] = \frac{1}{2\pi\theta}\delta^2(k)\,, \tag{2.9}$$

The D2-brane charge density formula (2.3) is evaluated as

$$J_{012}(x) = \mathcal{T}_{\rm D2}\Pi_{i=3}^{9}\delta(x_i)\,. \tag{2.10}$$

Here we used a relation among the tensions, $\mathcal{T}_{\rm D2} = \mathcal{T}_{\rm D0}/(4\pi^2\alpha')$. This (2.10), and the fact that other components of the D2-brane charge vanishes, explicitly shows that a flat D2-brane along x_1 and x_2 directions is generated by the algebra (2.8). This result was used for the analysis of Seiberg-Witten map [19].

In addition, we can compute the D0-brane charge density (2.1) as

$$J_0(x) = \frac{\mathcal{T}_{\rm D0}}{2\pi\theta}\Pi_{i=3}^{9}\delta(x_i)\,. \tag{2.11}$$

This means that smeared D0-branes bound on the generated D2-brane are present, and their density is given by $1/2\pi\theta$. From the (dual) ordinary picture of the D2-brane action, the induced D0-brane charge density can be seen in the RR coupling as $\mathcal{T}_{\rm D2} 2\pi\alpha' F_{12}$, where F_{12} is a constant gauge field strength on the flat D2-brane. This coincides with (2.11) through a famous relation [19] $F_{12} = 1/\theta$.

Here we have seen only the supergravity couplings, but in this case it was shown [2] that couplings to all the closed string excitations are coincident in the two pictures. In appendix A, we present an interesting realization of this democratic duality via non-BPS D1-branes.

3. Energetics of strings between branes

In this section, to illustrate the power of the multipole moments of the supergravity couplings in Matrix theory and in general nonabelian D-brane actions, we compute the energy-momentum tensor generated in the bulk space by the solutions of 1+1 and 1+3 dimensional nonabelian gauge theories. In [4], the bulk interpretations of the Yang-Mills solutions with nontrivial transverse scalar fields were given through the RR and NSNS B-field couplings generated in the bulk. The solutions we employ in this paper are — SU(2) solutions with or without preserving supersymmetries in 1+1 dimensions representing string networks [4] or recombined D-strings [6], and abelian/nonabelian BPS monopoles [4] and 1/4 BPS dyons [9, 10] in 1+3 dimensions. After showing the solutions and their RR and NSNS B-field couplings which were provided partly in [4], we compute the multipole moments in the coupling to the graviton to obtain the bulk distribution of the energy-momentum tensor generated by the matrix configurations.

3.1 Tachyon matter in Yang-Mills and D-brane recombination

The power of the multipole moments of the supergravity charge densities in Matrix theory and nonabelian D-brane actions becomes manifest when one considers condensation of off-diagonal components of the Yang-Mills fields. One of the examples in which the off-diagonal condensation has a significant geometric meaning was found in [6] where intersecting D-strings are recombined by the condensation. In this case the modes are tachyonic. Let us briefly look back this phenomena of the tachyon condensation described solely by the Yang-Mills theory. This 1+1 dimensional Yang-Mills theory has an SU(2) gauge group for the two D-strings, with its matter content consisting of a gauge field $A_\mu = A_\mu^a(\sigma_a/2)$ $(a = 1,2,3)$ and a scalar field[1] Y. The classical solution representing the D-strings intersecting at an angle θ is

$$Y = \tan\left(\frac{\theta}{2}\right) x\sigma_3 \, , \qquad A_\mu = 0 \, , \tag{3.1}$$

where $x \equiv x^1$ is a worldvolume spatial direction. Note that $\theta = 0$ corresponds to a parallel BPS pair of D-strings while $\theta = \pi$ represents an anti-parallel pair of D-strings (parallel

[1]This Y is normalized as $Y = 2\pi\alpha'\Phi$ where Φ is a usual YM scalar field, so that Y represents a target space distance. We turn on only a single scalar field among 8, for the present purpose.

brane-antibrane). The fluctuation eigen modes obtained in the Yang-Mills theory[2] include the following tachyon of the gaussian profile in x,

$$Y(x) = 2\pi\alpha' C(t) \exp\left[\frac{m^2x^2}{2}\right]\left(\frac{\sigma_1}{2}\right), \quad A_x(x) = C(x^0)\exp\left[\frac{m^2x^2}{2}\right]\left(\frac{\sigma_2}{2}\right). \tag{3.2}$$

Note that m^2 is negative here and in the following. The fluctuation field $C(t)$ obeys a free tachyon propagation equation $(\partial_t^2 + m^2)(t) = 0$ where the tachyonic mass squared is $m^2 = -\frac{1}{\pi\alpha'}|\tan(\theta/2)| < 0$, which coincides[3] with a string worldsheet spectrum given in [21] for a small θ. One of the solutions of the tachyon propagation equation is a "half S-brane"

$$C(t) = Ce^{\sqrt{-m^2}t}. \tag{3.3}$$

This is an analogue of the half S-brane background considered in [22], and in our context it means a classical evolution (recombination) starts at the past infinity when the D-strings are intersecting. The other solutions include a "full S-brane" [23] $C(t) = C\sinh(\sqrt{-m^2}t)$ in which the D-strings touch once at $t = 0$, and also an analogue of the rolling tachyon $C(t) = C\cosh(\sqrt{-m^2}t)$ considered originally in [24], for which the D-strings never touch each other. One can also consider a quantum treatment of this development of the condensation as in [7].

Let us study the bulk charges generated by this matrix configuration with the tachyon condensation $C(t)$. The bulk charge density formulas we need can be obtained by making a T-duality on the formulas given in section 2. For the RR two-form field, the formulas we need are

$$\begin{aligned}\widetilde{J}_{01} &= \mathcal{T}_{\rm D1}\,\mathrm{Str}\left[e^{ik_yY}\right], \\ \widetilde{J}_{0y} &= \mathcal{T}_{\rm D1}\,\mathrm{Str}\left[e^{ik_yY}D_1Y\right],\end{aligned} \tag{3.4}$$

where we give a Fourier transform in which k_y is the momentum along the y direction, and Str is the symmetrized trace, where Y, D_1Y (and F_{01} in the later examples) are treated as units in the symmetrization. For the NSNS B-field,

$$\widetilde{I}_{01}(x,k_y) = \mathcal{T}_{\rm D1}(2\pi\alpha')\,\mathrm{Str}\left[e^{ik_yY}F_{10}\right], \tag{3.5}$$

$$\widetilde{I}_{0y}(x,k_y) = \mathcal{T}_{\rm D1}(2\pi\alpha')\,\mathrm{Str}\left[e^{ik_yY}F_{10}D_1Y\right]. \tag{3.6}$$

A straightforward calculation shows that [4] the NSNS B-field coupling vanishes in the present solution with the tachyon condensation, and

$$J_{01}(x,y) = \mathcal{T}_{\rm D1}\left[\delta(y-\lambda(x)) + \delta(y+\lambda(x))\right],$$

[2] A fluctuation analysis for a T-dual configuration was provided in [20].

[3] In fact, the fluctuation spectrum obtained in this way in 1+1 dimensional Yang-Mills theory with 8 transverse scalar fields precisely coincides with the low-lying part of the worldsheet spectrum. It was pointed out in [6] that the first massive mode with $m^2 = \theta/2\pi\alpha'$ does not appear in the fluctuation of Y nor A_1. It looks that this contradicts with the worldsheet spectrum, but this is not the case. The fluctuation analysis of the remaining 7 transverse scalar fields provide 7 complex eigen modes at this mass level, which coincides with the worldsheet results in [21]. In the same manner, the number of the fluctuations at each mass level can be shown to be identical with that of corresponding string excitations. We thank S. Nagaoka for bringing us this point.

$$J_{0y}(x,y) \;=\; \mathcal{T}_{\rm D1}\lambda'(x)\left[\delta(y-\lambda(x))-\delta(y+\lambda(x))\right], \tag{3.7}$$

where $\lambda(x)\equiv\pi\alpha'\sqrt{m^4x^2+C(t)^2e^{m^2x^2}}$ is the magnitude of the eigenvalues of Y. Therefore the intersecting D-strings are recombined [6] and its separation is given by the magnitude of the tachyon condensation, $2\pi\alpha' C(t)$.

Let us turn our eyes to the graviton coupling which is given by making the T-duality,

$$\widetilde{T}_{00}(x,k_y) \;=\; \mathcal{T}_{\rm D1}\,{\rm Str}\left[e^{ik_yY}\left(\mathbb{1}+\frac{1}{2}(D_0Y)^2+\frac{1}{2}(D_1Y)^2+\frac{1}{2}(2\pi\alpha')^2F_{01}^2\right)\right], \tag{3.8}$$

$$\widetilde{T}_{11}(x,k_y) \;=\; \mathcal{T}_{\rm D1}\,{\rm Str}\left[e^{ik_yY}\left(-\mathbb{1}+\frac{1}{2}(D_0Y)^2+\frac{1}{2}(D_1Y)^2-\frac{1}{2}(2\pi\alpha')^2F_{01}^2\right)\right], \tag{3.9}$$

$$\widetilde{T}_{yy}(x,k_y) \;=\; \mathcal{T}_{\rm D1}\,{\rm Str}\left[e^{ik_yY}\left(\frac{1}{2}(D_0Y)^2-\frac{1}{2}(D_1Y)^2\right)\right]. \tag{3.10}$$

In the expression we neglected higher order terms in fields since we are working in Yang-Mills truncation. So we understand that the above formulas are reliable when these corrections are small. The order estimation [6] shows that we have to work with

$$C^2 \ll |m^2|\,. \tag{3.11}$$

With the tachyon condensation of the half S-brane (3.3), this means that there is a limitation on t beyond which the Yang-Mills analysis is not reliable.

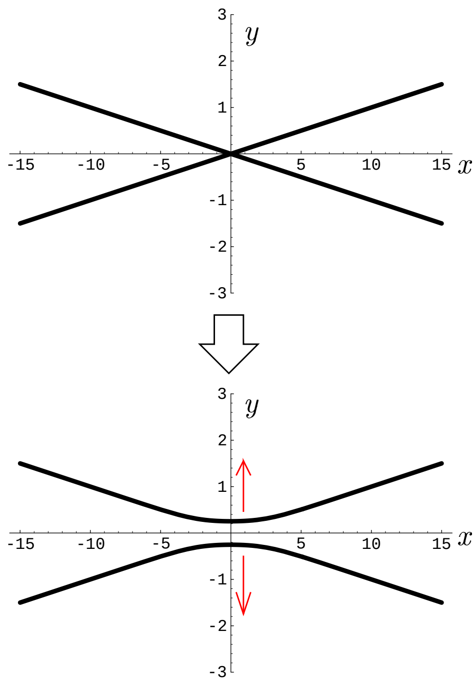

Figure 1: The intersecting D-strings are recombined by the tachyon condensation. The small arrows indicate the motion of the D-strings.

In [7], it was argued that in the recombination a part of the energy in Yang-Mills may remain in the region between the recombined D-strings. This is due to the non-vanishing energy originating in the off-diagonal gauge fields in the gauge diagonalizing Y. The energy was interpreted to be due to creation of a bunch of F-string-anti-F-string pairs connecting the upper and the lower D-strings.[4] (Since the asymptotic F-string charge bound on the recombined D-strings should vanish, the total number of created F-string charge should be zero, which means that created string should appear as a pair of F-string-anti-F-string.) This is a quite interesting phenomena, and here we apply the supergravity charge density formulas and confirm explicitly the presence of the string-like energy between the recombined D-strings. Substitution of our Yang-Mills configuration into the above formulas for the energy-momentum tensor gives

$$T_{\mu\nu} = T^{(1)}_{\mu\nu}\left(\delta(y-\lambda)+\delta(y+\lambda)\right)+T^{(2)}_{\mu\nu}\left(\theta(y+\lambda)-\theta(y-\lambda)\right), \tag{3.12}$$

[4]The existence of the strings connecting the recombined D-branes was observed in [25].

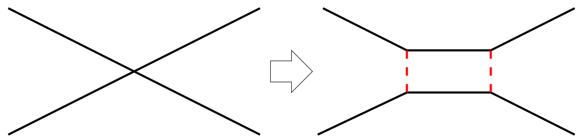

Figure 2: A schematic view of the created strings (dashed lines) connecting the recombined D-strings (solid lines). In the actual Yang-Mills configuration the dashed lines are smeared horizontally, while here for the simplified calcuations presented in the text we draw localized vertical strings.

where $\theta(z)$ is a step function which has a support only for $z > 0$ as $\theta(z > 0) = 1$. The coefficients are given by

$$\frac{T_{00}^{(1)}}{\mathcal{T}_{\rm D1}} = 1 + \frac{(\pi\alpha')^4}{2\lambda^2}\left(C^2\dot{C}^2e^{2m^2x^2} + m^4x^2(m^2 + C^2e^{m^2x^2})^2\right) \tag{3.13}$$

$$\frac{T_{11}^{(1)}}{\mathcal{T}_{\rm D1}} = -1 + \frac{(\pi\alpha')^4}{2\lambda^2}\left(C^2\dot{C}^2e^{2m^2x^2} + m^4x^2(m^2 + C^2e^{m^2x^2})^2\right) \tag{3.14}$$

$$\frac{T_{yy}^{(1)}}{\mathcal{T}_{\rm D1}} = \frac{(\pi\alpha')^4}{\lambda^2}\left(C^2\dot{C}^2e^{2m^2x^2} - m^4x^2(m^2 + C^2e^{m^2x^2})^2\right) \tag{3.15}$$

$$\frac{T_{00}^{(2)}}{\mathcal{T}_{\rm D1}} = \frac{(\pi\alpha')^4}{2\lambda^3}e^{m^2x^2}\left(2m^4x^2\dot{C}^2 + C^2\dot{C}^2e^{m^2x^2} + C^2(m^2 + C^2e^{m^2x^2})^2\right) \tag{3.16}$$

$$\frac{T_{11}^{(2)}}{\mathcal{T}_{\rm D1}} = \frac{(\pi\alpha')^4}{2\lambda^3}C^2e^{m^2x^2}\left(-\dot{C}^2e^{m^2x^2} + (m^2 + C^2e^{m^2x^2})^2\right) \tag{3.17}$$

$$\frac{T_{yy}^{(2)}}{\mathcal{T}_{\rm D1}} = \frac{(\pi\alpha')^4}{\lambda^3}e^{m^2x^2}\left(m^4x^2\dot{C}^2 - C^2(m^2 + C^2e^{m^2x^2})^2\right) \tag{3.18}$$

where the dot denotes a derivative with respect to time t. The important fact is that we have found a nonzero value of $T_{\mu\nu}^{(2)}$. In fact, $T_{00}^{(2)}$ is the energy found between the recombined D-strings.[5] We find two more facts which support that this energy is due to the creation of F-string pairs connecting the upper and the lower recombined D-strings: first, the energy $T_{00}^{(2)}$ is homogeneous along the y direction, and second, the pressure along y appears to be mostly negative, $T_{yy}^{(2)} < 0$, while there is a positive pressure along x^1, $T_{11}^{(2)} > 0$. The second fact may depend on the initial condition of the tachyon condensation, however if we employ a typical case of the half S-brane (3.3) we can show that this is the case in the valid region of the rolling tachyon. These strongly support the identification that the created energy is due to some string-like object oriented along y and connecting the two D-strings, with no net charge.

In fact, there is a reason why these strings should have appeared between the recombined D-strings. Imagine the situation in which these strings do not appear in the recombination. Then, the intersecting D-branes can lower their energy by recombining and separating from each other. The energy of them entirely comes from the lengths of the recombined D-strings. To evaluate this energy, we assume the shape of the recombined D-strings to be the solid lines in figure 2. Defining the distance between the recombined D-strings as $2\pi\alpha' C$, the energy gain due to the recombination can be calculated as

$$\delta E = -T_{\rm D1}\pi\alpha' C\theta\,. \tag{3.19}$$

[5]The result we find here for $T_{00}^{(2)}$ coincides with the number density of F-strings obtained in [7].

Note that this is linear in C. This means that the instability of the intersecting D-strings is the first order, that is, the pair of the intersecting D-branes is not a meta-stable vacuum in string theory. If this were true, one cannot define the mass spectrum around this configuration since this configuration is classically unstable. However from the worldsheet analysis we know that the intersecting D-branes are meta-stable vacua, hence we have to cancel this energy δE somehow. In fact, this energy linear in C can be cancelled by introducing a string-like "bond" between the recombined D-branes, as in the dashed lines in figure 2. The total tension of this bond should be

$$\mathcal{T} = \frac{|\delta E|}{2\pi\alpha' C} = \left(\frac{\theta}{2}\right) T_{\rm D1} \,. \tag{3.20}$$

This is the total tension of the effective strings expected from this simple argument of the meta-stable vacuum. Surprisingly, one can show that the precise Yang-Mills result for $T_{00}^{(2)}$ (3.16) reproduces this effective string tension,

$$\lim_{C\to 0} \int dx T_{00}^{(2)} = \left(\frac{\theta}{2}\right) T_{\rm D1} \,. \tag{3.21}$$

To show this explicitly, we have used the half S-brane background (3.3). The integrand in the left hand side of this equation is a delta function $\delta(x)$ in the limit $C \to 0$, which means that the created strings are localized at the intersection point before the tachyon condensation. These strings play the role of the "bond" which prevents recombined D-strings from coming apart.

Once the pairs of F-strings are created between the recombined D-strings, they may decay to something since they are unstable. However we are working in Yang-Mills theory where the coupling to the closed strings is neglected, so the energy would remain there. This leads us to the expectation that the end product should be a tachyon matter [8], since as studied in detail in [6], the recombination of the intersecting D-strings is a local version of the tachyon condensation of a brane-antibrane. The tachyon matter was originally considered as an end-product of a homogeneous tachyon condensation on a brane-antibrane or a non-BPS brane [8]. It is quite intriguing that we can observe the tachyon matter even in the scheme of Yang-Mills theory. In fact, we can give an evidence for that the end product is the tachyon matter. The important feature of the tachyon matter is that it is pressure-less in the far future ($t \to \infty$) while it preserves a nonzero energy density. In our case we cannot follow the large t behaviour in the Yang-Mills theory, but we may work in the region where (3.11) is satisfied. The energy is now distributed in the two-dimensional region between the D-strings, especially has a dependence on the x^1 direction. In the correspondence to the brane-antibrane where $\theta = \pi$, this x^1 direction would shrink effectively [6] and thus to see the quantity corresponding to the brane-antibrane picture we may integrate over the x^1 direction. Interestingly, the energy $T_{00}^{(2)}$ integrated over x^1 looks conserved, as in the case of the rolling tachyon on the brane-antibrane, see figure 3.[6] On the other hand, the magnitude of the pressure along the y direction which is the worldvolume

[6]In this numerical evaluation we took the region $t < 0$, since at this boundary $t = 0$ one has $|C^2/m^2| = 0.1$ and beyond this point our Yang-Mills analysis may be violated.

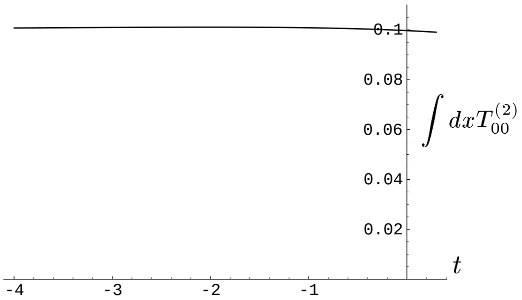

Figure 3: The energy resides between the recombined D-strings, integrated over x^1. We took $m^2 = -0.1$ and a half S-brane $C(t) = 0.1 e^{\sqrt{-m^2}t}$ in the unit $\pi\alpha' = 1$.

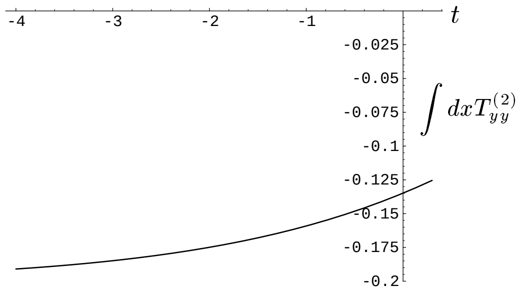

Figure 4: The pressure $T^{(2)}_{yy}$ between the recombined D-strings, integrated over x^1, with the same parameters. Its magnitude tends to decrease.

direction of the brane-antibrane, $T^{(2)}_{yy}$, has a tendency to decrease as time develops, after the integration over x^1 (see figure 4). So the Yang-Mills result is consistent with the brane-antibrane picture, and we conclude that Yang-Mills analysis describing the recombination of the D-strings captures also the property of the tachyon matter.

The energy $T^{(1)}_{00}$ is localized on the recombined D-strings, and it provides a time-dependent tension of them. Since we know the motion of the D-strings which is given by $y = \lambda(t, x)$, let us compare this $T^{(1)}_{00}$ with the energy of a single moving D-string whose location is specified by λ. The hamiltonian of a single D-string is given by

$$H_{\rm D1} = \mathcal{T}_{\rm D1} \frac{1 + \lambda'^2}{\sqrt{1 - \dot{\lambda}^2 + \lambda'^2}} = \mathcal{T}_{\rm D1} \left[1 + \frac{1}{2}\lambda'^2 + \frac{1}{2}\dot{\lambda}^2 + \text{higher terms} \right], \tag{3.22}$$

where the dash denotes a derivative with respect to $x \equiv x^1$. The "higher terms" consist of the Born-Infeld corrections and higher derivative corrections in the abelian D1-brane action. Interestingly, one can show that this hamiltonian coincides with the energy-momentum tensor found above,

$$H_{\rm D1} = T^{(1)}_{00} \,. \tag{3.23}$$

This is a strong support for that the energy-momentum tensor formulas (3.10) provide a correct distribution of the energy in the bulk for the recombination of the D-strings.

3.2 Energetics of a string network

A solution representing a supersymmetric recombination of (p, q)-strings was found in [4] where, by calculating the distributions of the D-string and fundamental string charges in the bulk, it was found that the fundamental strings connect the recombined D-strings. This is a generalization of the string network, and a naive estimation of the force balance was given there. Here we explicitly evaluate the energy-momentum tensor generated by this solution and show the stability and the BPS bound. The supersymmetric solution in 1+1 dimensional Yang-Mills theory is quite simple [4],

$$Y = 2\pi\alpha' A_0 = px\sigma_3 + a\sigma_1 \,, \qquad A_1 = 0 \,, \tag{3.24}$$

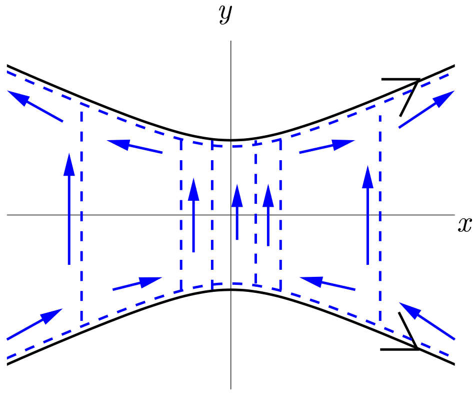

Figure 5: Charge distributions in the bulk produced by the Yang-Mills solution representing a string network. Arrows indicate the directions of the flux for the fundamental strings. The dashed lines denote the charge itself. Note that the vertical dashed lines are actually smeared along x although they look localized in this figure. Solid lines denote the (p,q)-strings.

where p and a are constant parameters which can be taken to be non-negative without losing generality. The generated F-string charge is given as [4]

$$I_{01}(x,y) = \mathcal{T}_{\rm D1}\lambda'\left[\delta(y-\lambda)-\delta(y+\lambda)\right], \tag{3.25}$$

$$I_{0y}(x,y) = \mathcal{T}_{\rm D1}\lambda'^2\left[\delta(y-\lambda)+\delta(y+\lambda)\right]+\mathcal{T}_{\rm D1}\lambda''\left[\theta(y+\lambda)-\theta(y-\lambda)\right]. \tag{3.26}$$

while the D-string charge has the same expression as (3.7) except that now the function specifying the location $\lambda(x)$ is defined as $\lambda(x)\equiv\sqrt{p^2x^2+a^2}$. The step function term in (3.26) shows the presence of the F-string directed along y direction and connecting the recombined (p,q)-strings. The configuration is shown in figure 5.

A simple calculation shows that the bulk energy-momentum tensor is generated as

$$T_{00} = \mathcal{T}_{\rm D1}(1+\lambda'^2)\left(\delta(y-\lambda)+\delta(y+\lambda)\right)+\mathcal{T}_{\rm D1}\lambda''^2\left(\theta(y+\lambda)-\theta(y-\lambda)\right), \tag{3.27}$$

$$T_{11} = -\mathcal{T}_{\rm D1}\left(\delta(y-\lambda)+\delta(y+\lambda)\right), \tag{3.28}$$

$$T_{yy} = -\mathcal{T}_{\rm D1}\lambda'^2\left(\delta(y-\lambda)+\delta(y+\lambda)\right)-\mathcal{T}_{\rm D1}\lambda''^2\left(\theta(y+\lambda)-\theta(y-\lambda)\right). \tag{3.29}$$

The first term in T_{00} describes the tension of the recombined (p,q)-strings. Noting that the expression is provided with the delta function of y, we take into account the slope λ' and find that the tension of it is $\mathcal{T}_{\rm D1}\sqrt{1+\lambda'^2}$. This in fact saturates the BPS bound, since the F-string density vector derived from (3.25) and the first terms in (3.26) is $\mathcal{T}_{\rm D1}\lambda'\vec{n}$ where $\vec{n}$ is the unit vector along the trajectory of the (p,q)-string (we took into account the slope factor again). The second term in T_{00} is the tension of the F-string connecting the recombined (p,q)-strings, and again saturates the BPS bound since it coincides with the density of the connecting F-strings provided in the second term in (3.26). The orientation of all of these strings are found to be consistent with the bulk supersymmetry of string networks studied in [26].

Let us check the force balance from the pressure terms (3.28) and (3.29). First, the force along the x direction exists only on the recombined (p,q)-strings since there is no step function term in (3.28). This is consistent with the fact that the connecting F-strings are oriented along y direction. The net force along the x direction is given by

$$\int_{x=\bar{x}} dy\, T_{11} = -2\mathcal{T}_{\rm D1} \tag{3.30}$$

which is constant and does not depend on a slice $\bar{x}$. The support of the integrand is a delta function whose location is just at the intersection of the D-strings and the integration slice. This means that the force along the x direction is balanced. The more nontrivial check is on the force along y. We find

$$\int_{y=\bar{y}} dx\, T_{yy} = -2p\mathcal{T}_{\rm D1} \tag{3.31}$$

and this is independent of x, which shows the force balance. Note that the net force is carried only by the connecting F-strings if $|\bar{y}| < a$ while only by the (p,q)-strings if $|\bar{y}| \to \infty$. So the force is transfered from one to another when one changes the slice $\bar{y}$, while the net force is kept fixed. This is the same as the well-known stability of BPS string networks.

The force balance condition is intimately related to the current conservation of the energy-momentum tensor. For any solution of Matrix theory, it was shown that the current conservation for bulk currents including the energy momentum tensor holds [27]. Our system is almost T-dual to the Matrix theory (D0-brane effective theory), thus in our case we may expect the current conservation. For example, we have

$$\partial_y T_{yy} + \partial_1 T_{1y} - \partial_0 T_{0y} = 0\,. \tag{3.32}$$

Noting that the configuration is static ($\partial_0 T_{\mu\nu} = 0$), we integrate this expression over x and obtain

$$\partial_y \int T_{yy} dx = 0\,. \tag{3.33}$$

We assumed that there is no surface term for T_{1y}. The result is consistent with the equation (3.31). However, we find that this current conservation condition (3.33) is not so strong to give the force balance, since (3.33) means just that $\int dx T_{yy}$ is independent of y and thus says that it may depend on x. The actual force balance is established by showing that the quantity such as $\int dx T_{yy}$ is constant for each connected distribution of the energy-momentum in the bulk.

3.3 The shape of monopoles

In [4], the RR charge distributions of the BPS monopole configurations in 1+3 dimensional U(1) and SU(2) Yang-Mills theory were computed, and in the SU(2) case the existence of D-strings extending out of the D3-brane worldvolume has been found. Here we employ the same classical solutions and evaluate the energy-momentum tensors generated in the bulk, to see the force balance and the consistency with the RR densities found in [4].

The formulas we need here are obtained by taking T-dualities third times on the formulas for D0-branes. The result for the case $D_0 Y = F_{0i} = 0$ is

$$\widetilde{T}_{00}(x_i, k_y) = \mathcal{T}_{\rm D3}\, {\rm Str}\left[e^{ik_y Y}\left(\mathbb{1} + \frac{1}{2}(D_i Y)^2 + \frac{1}{4}(2\pi\alpha')^2 F_{ij}^2\right)\right], \tag{3.34}$$

$$\widetilde{T}_{yy}(x_i,k_y) = -\mathcal{T}_{\rm D3}\,{\rm Str}\left[e^{ik_yY}(D_iY)^2\right], \tag{3.35}$$

$$\begin{aligned}\widetilde{T}_{11}(x_i,k_y) = \mathcal{T}_{\rm D3}\,{\rm Str}\Bigg[e^{ik_yY}\bigg(-\mathbb{1}+\frac{1}{2}\left((D_1Y)^2-(D_2Y)^2-(D_3Y)^2\right)\\ +\frac{1}{2}(2\pi\alpha')^2\left(F_{12}^2+F_{13}^2-F_{23}^2\right)\bigg)\Bigg],\end{aligned} \tag{3.36}$$

where $i=1,2,3$ is the worldvolume coordinates of the D3-brane(s). Similar formulas can be obtained for T_{22} and T_{33}. In the examples we use in the following, the BPS equation

$$2\pi\alpha'\epsilon_{ijk}F_{ijk} = 2D_iY \tag{3.37}$$

holds, and the above formulas reduce to

$$\begin{aligned}\widetilde{T}_{00}(x_i,k_y) &= \mathcal{T}_{\rm D3}\,{\rm Str}\left[e^{ik_yY}\left(\mathbb{1}+(D_iY)^2\right)\right],\\ \widetilde{T}_{yy}(x_i,k_y) &= -\mathcal{T}_{\rm D3}\,{\rm Str}\left[e^{ik_yY}(D_iY)^2\right],\\ \widetilde{T}_{11}(x_i,k_y) &= \widetilde{T}_{22}(x_i,k_y)=\widetilde{T}_{33}(x_i,k_y) = -\mathcal{T}_{\rm D3}\,{\rm Str}\left[e^{ik_yY}\right].\end{aligned} \tag{3.38}$$

First we deal with the abelian monopole (BIon) [28],

$$B^i = \frac{1}{2}\epsilon_{ijk}F_{jk} = \frac{-bx_i}{r^3}, \qquad Y = 2\pi\alpha'\frac{b}{r}, \qquad r\equiv\sqrt{(x_1)^2+(x_2)^2+(x_3)^2}\,. \tag{3.39}$$

The generated D-string density is [4]

$$J_{0y} = \mathcal{T}_{\rm D3}(2\pi\alpha')^2\frac{b^2}{r^4}\delta\left(y-\frac{2\pi\alpha' b}{r}\right), \tag{3.40}$$

$$J_{0i} = -\mathcal{T}_{\rm D3}(2\pi\alpha')\frac{bx_i}{r^3}\delta\left(y-\frac{2\pi\alpha' b}{r}\right), \tag{3.41}$$

which is depicted in figure 6. Since the theory is abelian, the location of the D3-brane charge is trivially determined by the value of Y. The computation of the energy-momentum tensor (3.38) results in

$$T_{00} = \mathcal{T}_{\rm D3}\left(1+(2\pi\alpha')^2\frac{b^2}{r^4}\right)\delta\left(y-\frac{2\pi\alpha' b}{r}\right), \tag{3.42}$$

$$T_{yy} = -\mathcal{T}_{\rm D3}(2\pi\alpha')^2\frac{b^2}{r^4}\delta\left(y-\frac{2\pi\alpha' b}{r}\right), \tag{3.43}$$

$$T_{11} = T_{22} = T_{33} = -\mathcal{T}_{\rm D3}\delta\left(y-\frac{2\pi\alpha' b}{r}\right). \tag{3.44}$$

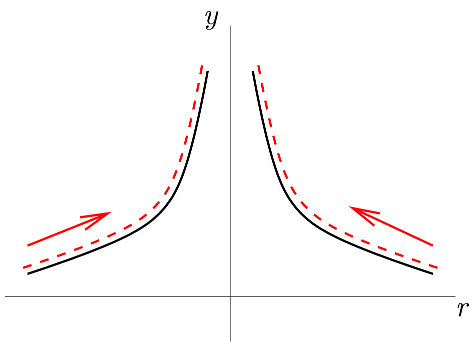

Figure 6: Charge distribution in the BIon. The dashed lines with arrows are the D-string charges bound on the D3-brane spike (solid lines).

As described in the previous subsection, the force balance can be seen in the integration of the energy-momentum tensor. That for the x_i directions is trivial,

$$\int_{x_1=\bar{x}_1} dy dx_2 dx_3\, T_{11} = -\mathcal{T}_{\rm D3}V_{23} \tag{3.45}$$

where V_{23} is the volume of the x^2 and x^3 direction. This is independent of x^1 which shows the force balance along x^1. A physically important check is the force balance along y, which can be shown as

$$\int_{y=\bar{y}} dx_1 dx_2 dx_3\, T_{yy} = -4\pi \mathcal{T}_{\rm D3} 2\pi\alpha' b = -n\mathcal{T}_{\rm D1} \tag{3.46}$$

which coincides with the tension of n D-strings where the number of the D-strings has been quantized as $n = b/2$. One can show that the T_{00} saturates a BPS bound of the D1-D3 bound state and also that the orientation of the surface is consistent with the bulk supersymmetry, as in the same way as the string network of the previous section.

The power of the multipole moment formulas becomes manifest when we consider nonabelian monopoles. The SU(2) 'tHooft-Polyakov BPS monopole solution is [29]

$$A_i = \epsilon_{aij}(1-K(r))\frac{x_j}{r^2}\left(\frac{\sigma_a}{2}\right), \qquad A_0 = 0\,, \qquad Y = -2\pi\alpha' H(r)\frac{x_i}{r^2}\left(\frac{\sigma_i}{2}\right),$$

where $K(r) \equiv Pr/\sinh(Pr)$, $H(r) \equiv Pr\coth(Pr) - 1$, and P is a constant. In [30], this configuration was interpreted to be a D-string suspended between parallel D3-branes separated by $2\pi\alpha' P$, through the matching of the supersymmetry, the total energy and the RR charges. Here we can use the charge density formulas to evaluate the actual bulk distribution of the energy and the charges. Using (3.38), we obtain

$$\begin{aligned}
\frac{T_{00}}{\mathcal{T}_{\rm D3}} &= \left(1 + (2\pi\alpha')^2\frac{(1-K^2)^2}{4r^4}\right)(\delta(y-\lambda)+\delta(y+\lambda)) + 2\pi\alpha'\frac{HK^2}{r^3}(\theta(y+\lambda)-\theta(y-\lambda))\,,\\
\frac{T_{11}}{\mathcal{T}_{\rm D3}} &= \frac{T_{22}}{\mathcal{T}_{\rm D3}} = \frac{T_{33}}{\mathcal{T}_{\rm D3}} = -\left(\delta(y-\lambda)+\delta(y+\lambda)\right),\\
\frac{T_{yy}}{\mathcal{T}_{\rm D3}} &= -(2\pi\alpha')^2\frac{(1-K^2)^2}{4r^4}(\delta(y-\lambda)+\delta(y+\lambda)) - 2\pi\alpha'\frac{HK^2}{r^3}(\theta(y+\lambda)-\theta(y-\lambda))\,.
\end{aligned} \tag{3.47}$$

The D-string charge distribution was computed in [4],

$$J_{0y} = -T_{yy}\,, \qquad J_{0i} = (2\pi\alpha')\mathcal{T}_{\rm D3}\frac{x_i}{2r^3}(1-K^2)\left(\delta(y-\lambda)-\delta(y+\lambda)\right). \tag{3.48}$$

Here λ denotes the location of the D3-brane which is specified by the eigenvalues of Y, $\pm y = -\lambda(x_1,x_2,x_3) \equiv \pi\alpha' H(r)/r$. The D-string charge distribution is depicted in figure 7. As for the first term in T_{00}, one can show in a similar way that this provides a BPS mass bound where the number of bound D-strings can be found in the J^{0i} above. The second term, which exists only between the two D3-branes, is consistent with the D-string distribution since it coincides with the second term in J^{0y}. Therefore this configuration is supersymmetric in the bulk.

Let us compute the force balance. Along the y direction, we find

$$\int_{y=\bar{y}} d^3x\, T_{yy} = -\mathcal{T}_{\rm D1} \tag{3.49}$$

for any slice with fixed y. Therefore, we conclude that the force is balanced. To show this the presence of the bulk D-strings are necessary, thus the bulk D-strings are pulling the D3-brane surfaces to make them in the shape determined by the eigenvalues of the field Y.

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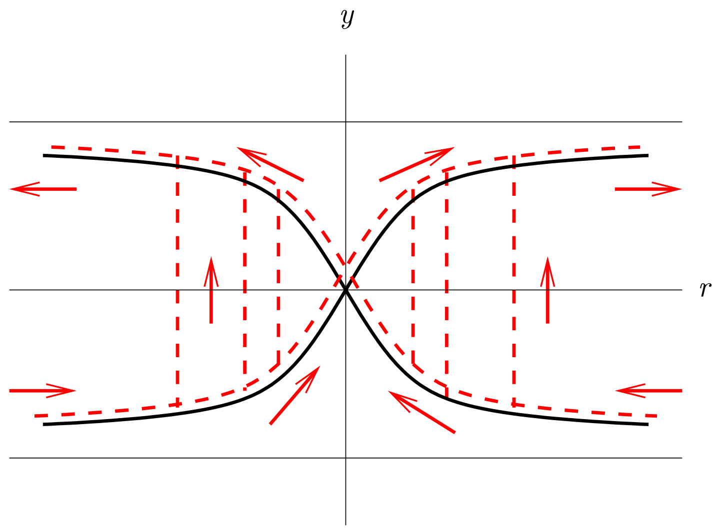

Figure 7: D-string charge distribution (dashed lines) of the 'tHooft-Polyakov monopole. D-strings away from the D3-brane surface connect the upper and the lower D3-branes.

An additional observation with the above equation is that the total pressure is equal to that of a single D-string. Hence one can say that if we neglect the detailed structure around the origin, macroscopically this configuration is equivalent to a D-string suspended between two parallel D3-branes as interpreted in [30]. Let us study this in detail.[7] With the slice $y = 0$, all the D-string charge in this configuration is provided by the bulk D-strings, that is, the second terms in (3.47). The distribution of the bulk D-string is given by the number density, $2\pi\alpha' HK^2/r^3$. This function has a single peak at $r = 0$ and it damps for large r as e^{-2Pr}. This means that, when the asymptotic separation of the D3-brane $(= 2\pi\alpha' P)$ is large compared to the string scale ($\Leftrightarrow P \gg 1/l_{\rm string}$), the bulk D-string charge is distributed within a tube with a radius smaller than the string scale. This is consistent with [30] in that for the large separation the detailed structure of the distributed D-strings in the bulk is hidden in the region within the distance of the string length from the D-brane surfaces. If we take $P \to \infty$ limit (the large separation between the asymptotically parallel D3-branes), the bulk D-strings HK^2/r^3 disappear, which is consistent with the the abelian case we considered before.

3.4 The shape of 1/4 BPS dyons

In [9], a new solution of $\mathcal{N} = 4$ SU(3) Yang Mills theory in 1+3 dimensions was constructed. The solution preserves 1/4 of the original supersymmetries and possesses a dyonic charge where the electric charge is not parallel to the magnetic charge due to the inclusion of two transverse scalar fields Y and Z. The existence of this kind of solutions was conjectured in [31] from the observation that string networks can suspend among several parallel D3-branes in such a way that 1/8 of the bulk supersymmetries are preserved. This is a generalization of the interpretation of the 'tHooft-Polyakov monopole as a D-string suspended between parallel D3-branes in the previous subsection.

[7]We thank A. Hashimoto for a discussion on this point.

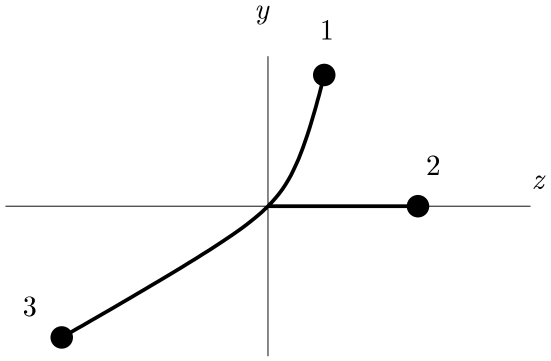

Figure 8: Location of the D3-brane surfaces of a 1/4 BPS dyon solution in SU(3), in course of varying r in the y-z space in the bulk (solid lines). We have chosen $(\alpha, \beta) = (-1, 1/3)$. Three blobs are the location of the D3-branes at $r = \infty$. They deform their shape to reach the origin $y = z = 0$ where the three D3-branes meet at $r = 0$.

In [9], it was found that the trajectories of the deformed D3-brane surfaces determined by the eigenvalues of Y and Z in the solution bend in a nontrivial manner in the bulk spacetime. The location of the D3-brane in course of varying r (the radial direction in the worldvolume) is depicted in figure 8. The way they are bent was found [10] to be consistent with the "effective string network". It is defined with the "effective (p, q)-strings whose (p, q) charges are defined by the electric and magnetic charges measured inside the sphere of the radius r in the worldvolume. The orientation of the effective (p, q)-strings are along the lines tangential to the D3-branes at r in the Y-Z plane in the bulk. Surprisingly, these effective (p, q)-strings form a stable consistent string network.

Here we evaluate the distribution of D-string and F-string charges in the bulk directly from the solution given in [9] to see the consistency with this picture of "effective" string network. The solution is spherically symmetric under a linear combination of the worldvolume and gauge rotations,

$$\epsilon_{ijk} x_j \partial_k + T_i \qquad (i = 1, 2, 3) \tag{3.50}$$

where the first term is a generator of the worldvolume rotation, while T_i is a generator of the maximal embedding of SU(2) in SU(3) with $T_3 = \mathrm{diag}(1, 0, -1)$. The solution (rescaled by a factor $2\pi\alpha'$ compared to that of [9]) is written on the x_3 axis as

$$Y = -2\pi\alpha' \frac{H(r)}{r} T_3 \,, \qquad Z = -A_0 = 2\pi\alpha' \frac{1}{4r} \mathrm{diag}(\varphi_+ + \varphi_-, -2\varphi_-, -\varphi_+ + \varphi_-)$$

where

$$\varphi_+ \equiv \alpha H(r) \,, \qquad \varphi_- \equiv \beta \left(3 \coth Pr - Pr \frac{2\cosh^2 Pr + 1}{\sinh^2 Pr} \right) , \tag{3.51}$$

$r \equiv \sqrt{\sum_{i=1}^{3} (x_i)^2}$, and α, β are constant parameters.

The D3-brane charge density is found to be

$$J_{0123} = \sum_{i=1,2,3} \delta(z - \lambda_i(r)) \delta(y - \lambda(r)) \,, \tag{3.52}$$

where λ_i's are the eigenvalues of the field Z while $\lambda \equiv 2\pi\alpha' H(r)/r$ is for Y,

$$\lambda_1(r) \equiv \pi\alpha' \frac{\varphi_+ + \varphi_-}{2r}, \qquad \lambda_2(r) \equiv -\pi\alpha' \frac{\varphi_-}{r}, \qquad \lambda_3(r) \equiv \pi\alpha' \frac{-\varphi_+ + \varphi_-}{2r}. \tag{3.53}$$

Here y and z are the bulk coordinates corresponding to the transverse scalars Y and Z respectively. The location of the D3-branes in the bulk is shown in figure 8. Due to the way of constructing the spherically symmetric solutions, Z and Y are simultaneously diagonalizable, and thus the location of the D3-brane is definite in the bulk. (In the next section, we shall treat the cases where two or more transverse scalars are noncommutative with each other.)

Since the solution satisfies the BPS equations $2\pi\alpha' B_i = D_i Y$ and $2\pi\alpha' E_i = D_i Z$, the D-string/F-string charge densities are given in the present case by

$$\begin{aligned}
\widetilde{J}_{0y} &= \mathcal{T}_{\mathrm{D3}} \,\mathrm{Str}\left[\exp\left[ik_z Z + ik_y Y\right] D_i Y D_i Y\right], \\
\widetilde{J}_{0z} = \widetilde{I}_{0y} &= \mathcal{T}_{\mathrm{D3}} \,\mathrm{Str}\left[\exp\left[ik_z Z + ik_y Y\right] D_i Z D_i Y\right], \\
\widetilde{I}_{0z} &= \mathcal{T}_{\mathrm{D3}} \,\mathrm{Str}\left[\exp\left[ik_z Z + ik_y Y\right] D_i Z D_i Z\right].
\end{aligned}$$

A straightforward but tedious calculations show

$$\begin{aligned}
\frac{J_{0y}}{\mathcal{T}_{\mathrm{D3}}} &= (\lambda')^2 \left[\delta(z-\lambda_1)\delta(y-\lambda) - \delta(z-\lambda_3)\delta(y+\lambda)\right] + \\
&\quad + \lambda \frac{2K^2(r)}{r^2} \Bigg[(\theta(y-\lambda)-\theta(y))\,\delta\left(\lambda_2 - z + y\frac{\lambda_1-\lambda_2}{\lambda}\right) + \\
&\qquad + (\theta(y+\lambda)-\theta(y))\,\delta\left(-\lambda_2 + z + y\frac{\lambda_3-\lambda_2}{\lambda}\right)\Bigg], \\
\frac{J_{0z}}{\mathcal{T}_{\mathrm{D3}}} &= \frac{I_{0y}}{\mathcal{T}_{\mathrm{D3}}} \\
&= \lambda' \left[\lambda_1'\delta(z-\lambda_1)\delta(y-\lambda) - \lambda_3'\delta(z-\lambda_3)\delta(y+\lambda)\right] + \\
&\quad + \frac{2K^2(r)}{r^2} \Bigg[(\lambda_1-\lambda_2)(\theta(y-\lambda)-\theta(y))\,\delta\left(\lambda_2 - z + y\frac{\lambda_1-\lambda_2}{\lambda}\right) + \\
&\qquad + (\lambda_2-\lambda_3)(\theta(y+\lambda)-\theta(y))\,\delta\left(-\lambda_2 + z + y\frac{\lambda_3-\lambda_2}{\lambda}\right)\Bigg], \\
\frac{I_{0z}}{\mathcal{T}_{\mathrm{D3}}} &= (\lambda_1')^2\delta(z-\lambda_1)\delta(y-\lambda) + (\lambda_2')^2\delta(z-\lambda_2)\delta(y) + (\lambda_3')^2\delta(z-\lambda_3)\delta(y+\lambda) + \\
&\quad + \frac{2K^2(r)}{r^2} \Bigg[\frac{(\lambda_1-\lambda_2)^2}{\lambda}(\theta(y-\lambda)-\theta(y))\,\delta\left(\lambda_2 - z + y\frac{\lambda_1-\lambda_2}{\lambda}\right) + \\
&\qquad + \frac{(\lambda_2-\lambda_3)^2}{\lambda}(\theta(y+\lambda)-\theta(y))\,\delta\left(-\lambda_2 + z + y\frac{\lambda_3-\lambda_2}{\lambda}\right)\Bigg].
\end{aligned}$$

We have checked that these components of the currents satisfy the current conservation condition, $\partial_z J_{0z} + \partial_y J_{0y} = 0$, $\partial_z I_{0z} + \partial_y I_{0y} = 0$. The fact we can find from these explicit expressions of the string current is that there are two (p,q)-strings suspended between the D3-branes, one is between the first and the second D3-brane located at $(z,y) = (\lambda_1, \lambda)$ and $(\lambda_2, 0)$, another is between the second and the third, $(\lambda_2, 0)$ and $(\lambda_3, -\lambda)$. These strings

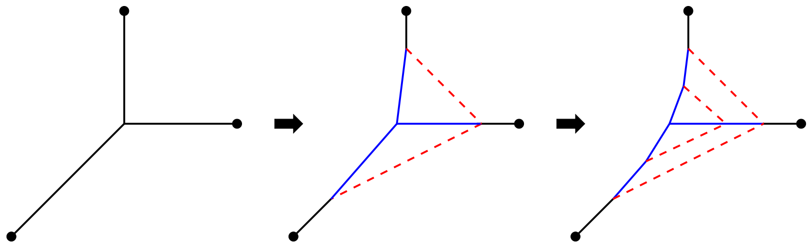

Figure 9: We show that the "effective string network" found in [10] is consistent with the bulk (p,q)-strings calculated here. At $r=\infty$, the lines tangential to the D3-brane (blobs) meet at a point (Left), which is an effective string network. The presence of (p,q) strings (dashed lines) in the bulk connecting two of three D3-branes make the D3-brane (solid lines) bend in the Z-Y plane (Right), in such a way that at each r the effective string network makes sense.

take a part of the electro-magnetic charges on the D3-branes out into the bulk, and change the shape of the D3-branes nontrivially. In fact, one can see that the first and the third D3-branes are pulled by these bulk strings and resultantly bend consistently in figure 8.

The "effective" string network at r in [10] was defined as a collection of "effective" (p,q)-strings whose charges are provided by the electric and magnetic charges integrated over a ball whose radius is r. Therefore, actual bulk (p,q) strings should appear as a difference between the effective string network at $r+\delta r$ and that at r. This is what we found, the strings connecting the first and the second, and the second and the third, D3-branes. A graphical description of this charge conservation is shown in figure 9.

Finally, the energy distribution in the bulk is given by

$$T_{00} = J_{0123} + I_{0z} + J_{0y}\,. \tag{3.54}$$

One can obtain similar expression for pressures and check that the energy-momentum tensor is conserved, and forces are balanced in the bulk.

4. Fuzzy S^2

So far, we have studied the D-brane/F-string charge distribution of nonabelian worldvolume configurations with a single transverse scalar fields. (The last example in section 3.4 uses two scalar fields but they are mutually commutative.) Here and in the following sections, we go into the examples with more intrinsic noncommutativity — two or more scalar fields are mutually noncommutative. We shall see how the nontrivial commutation relations among transverse scalar fields in D0-brane effective field theory give extended distribution of the charges in the bulk, thorough the supergravity charge density formulas.

In this section, we work with fuzzy two-sphere configuration of the matrix scalar fields in the D0-brane low energy effective theory. Spherical membranes in Matrix theory are known as one of the fundamental bound states formed by D0-branes [33]. Explicit configuration of the D0-brane matrices are given as

$$X_i = \frac{2R}{N} L_i \tag{4.1}$$

where L_i ($i = 1, 2, 3$) are an N-dimensional irreducible representation of generators of SU(2) satisfying $[L_i, L_j] = i\epsilon_{ijk} L_k$, so that the matrices X_i are subject to the algebra

$$[X_i, X_j] = \frac{2R}{N} i\epsilon_{ijk} X_k \,. \tag{4.2}$$

This matrix configuration is called a fuzzy S^2. This radius R can be seen in the relation

$$\sum_{i=1}^{3} X_i^2 = R^2 \left(1 - \frac{1}{N^2}\right) \mathbb{1}_N \,, \tag{4.3}$$

in which there appears a $1/N^2$ correction. One can absorb this correction factor into the redefinition of R, but we leave it as it is for simplicity of the calculations.

This locus equation (4.3) suggests that the D0-branes are distributed on a single S^2 uniformly. This expectation further leads us to a conjecture that the fuzzy sphere configuration defined by (4.1) is a dual (equivalent another) description of a spherical D2-brane on which N D0-branes are uniformly bound. This conjecture is a generalization of the fact observed in section 2 for the fuzzy plane. In the following subsections we shall see if this conjecture might come out to be true or not, by looking at various D-brane charge densities generated by the matrix configuration (4.1).

4.1 D-brane charge distribution

4.1.1 D0-brane charge

Let us evaluate the D0-brane charge density (2.1) for this fuzzy S^2 configuration of D0-branes (4.1). In the evaluation we may bring the direction of the vector $\vec{k}$ along x_3, without losing generality. This is due to the SU(2) $\sim$ SO(3) symmetry of the D0-brane configurations (4.1): for any given rotation matrix R of SO(3), there is an element of SU(2) which satisfies

$$R_{ij} X_j = U(R) X_i U(R)^\dagger \,. \tag{4.4}$$

Then the charge density becomes

$$\widetilde{J}_0(\vec{k}) = \mathcal{T}_{\mathrm{D0}} \;\mathrm{tr} \left[e^{ikX_3}\right] , \tag{4.5}$$

where $k \equiv \sqrt{k_1^2 + k_2^2 + k_3^2}$. Hereafter we omit trivial dependence on the other spatial directions x^μ with $\mu = 4, 5, \ldots, 9$. We may use a particular representation of the SU(2) generators in which the representation of L_3 becomes diagonal,

$$L_3 = \mathrm{diag}\left(\frac{N-1}{2}, \frac{N-3}{2}, \ldots, -\frac{N-1}{2}\right) . \tag{4.6}$$

With this representation the density (4.5) is evaluated as

$$\widetilde{J}_0(\vec{k}) = \mathcal{T}_{\mathrm{D0}} \sum_{m=-(N-1)/2}^{(N-1)/2} \exp\left[ikm\frac{2R}{N}\right] = \begin{cases} \mathcal{T}_{\mathrm{D0}} \displaystyle\sum_{n=1}^{N/2} \left(e^{ikR(2n-1)/N} + e^{-ikR(2n-1)/N}\right) & (N \text{ even}) \\ \mathcal{T}_{\mathrm{D0}} \left(1 + \displaystyle\sum_{n=1}^{(N-1)/2} \left(e^{2ikRn/N} + e^{-2ikRn/N}\right)\right) & (N \text{ odd}) \end{cases} \tag{4.7}$$

Here the summation for m is with a unit spacing.

To obtain the inverse Fourier transform of this expression, it is useful to find the Fourier transform of a delta function $\delta(r-a)$ where $r \equiv \sqrt{x_1^2+x_2^2+x_3^2}$,

$$\int d^3x\, e^{ik_ix_i}\delta(r-a) = \frac{-2\pi i a}{k}(e^{ika}-e^{-ika})\,, \tag{4.8}$$

and hence for $\delta'(r-a)$

$$\begin{aligned}\int d^3x\, e^{ik_ix_i}\delta'(r-a) &= -\frac{\partial}{\partial a}\int d^3x\, e^{ik_ix_i}\delta(r-a) \\ &= \frac{2\pi i}{k}(e^{ika}-e^{-ika}) - 2\pi a(e^{ika}+e^{-ika})\,. \end{aligned} \tag{4.9}$$

From these, we obtain a relevant formula for the inverse Fourier transform in three dimensions,

$$D(r,a) \equiv \frac{-1}{2\pi a}\left(\frac{1}{a}\delta(r-a)+\delta'(r-a)\right) \quad \xrightarrow{\text{F.T.}} \quad e^{iak}+e^{-iak}\,. \tag{4.10}$$

Using this formula, the inverse Fourier transform of the D0-brane density (4.7) is given by

$$J_0(x) = \begin{cases} \mathcal{T}_{\mathrm{D0}}\displaystyle\sum_{n=1}^{N/2} D(r,R^{(n)}) & (N \text{ even}) \\ \mathcal{T}_{\mathrm{D0}}\left(\delta^3(x)+\displaystyle\sum_{n=1}^{(N-1)/2} D(r,R^{(n)})\right) & (N \text{ odd}) \end{cases} \tag{4.11}$$

where $D(r,R^{(n)})$ is a spherical delta function (plus its derivative) with a support on a spherical shell whose radius $R^{(n)}$ $(n=1,2,\ldots,[N/2])$ is given by

$$R^{(n)} \equiv \begin{cases} \dfrac{2n-1}{N}R & (N \text{ even}) \\ \dfrac{2n}{N}R & (N \text{ odd}) \end{cases} \tag{4.12}$$

This means that the D0-branes are distributed on a collection of two-spheres (shells) with these different radii. Since we get an integration of each shell as

$$\int 4\pi r^2 dr\, D(r,R^{(n)}) = 2\,, \tag{4.13}$$

we find that each shell is composed of two D0-branes. When N is odd, the remaining single D0-brane is located at the origin.

The result that the D0-branes are distributed on a collection of shells seems quite strange.[8] Naively one expects from the matrix equation showing the spherical locus (4.3) that all N D0-branes would have been distributed on a single spherical shell with the radius R corrected by $1/N^2$. The fuzzy sphere, a bound state of D0-branes, given by (4.1) is expected to have a dual description by a single spherical D2-brane with magnetic field on it. However, the direct calculation of the D0-brane charge distribution shows that there

[8] M. van Raamsdonk has considered a related puzzle [32].

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are many spherical shells with various radii, on the contrary, and our naive expectation is not the case. Therefore, with use of the charge density formulas adopted in this paper, the duality for finite N does not seem to hold. In the next subsection we shall see that this discrepancy is resolved if we take the continuum (large-N) limit.

For a later convenience, here let us see a consistency of our computation. When $k_1 = k_2 = 0$, the D0-brane density for the fuzzy S^2 is given by

$$\widetilde{J}_0(k_1 = k_2 = 0, k_3) = \mathcal{T}_{\rm D0} \sum_{m=-(N-1)/2}^{(N-1)/2} \exp\left[ik_3 m \frac{2R}{N}\right]. \tag{4.14}$$

And the Fourier transform of this is given by

$$\int dx_1 dx_2\, J_0(\vec{x}) = \mathcal{T}_{\rm D0} \sum_{m=-(N-1)/2}^{(N-1)/2} \delta\left(z - \frac{2mR}{N}\right). \tag{4.15}$$

This seems to be contradictory to the result (4.11) since each shell consists of two D0-branes. Normally one may expect that for a uniformly distributed D0-branes on a shell, an integration over two dimensions orthogonal to the x_3 axis gives a uniform distribution of D0-branes on the axis, not the delta functions at the poles of the sphere. This apparent discrepancy can be resolved if one notes the second term in the density function $D(r, R^{(n)})$ (4.10). In fact, using polar coordinates, we can calculate for $N = 2$ the relevant quantity as

$$\begin{aligned}
\int dx_1 dx_2\, J_0(\vec{x}) &= \int 2\pi\rho d\rho\, J_0(\vec{x}) \\
&= \frac{-\mathcal{T}_{\rm D0}}{(R/2)^2} \int \rho d\rho \left(\delta\left(r - \frac{R}{2}\right) + \frac{R}{2}\delta'\left(r - \frac{R}{2}\right)\right) \\
&= \mathcal{T}_{\rm D0}\left(\delta\left(x_3 - \frac{R}{2}\right) + \delta\left(x_3 + \frac{R}{2}\right)\right),
\end{aligned} \tag{4.16}$$

where $\rho \equiv \sqrt{x_1^2 + x_2^2}$. So the strange term of the derivative of the delta function works for consistency.

4.1.2 D2-brane charge

To see what is a possible dual D2-brane description might be, let us evaluate the D2-brane charge density (2.3) generated by the fuzzy S^2 configuration (4.1). We note that from the rotational covariance of the formula (2.3) the final expression of this charge density should be of the form $\widetilde{J}_{0ij} = k_m \epsilon_{ijm} f(k)$ where $f(k)$ is some function of the total momentum k. Based on this fact, we may put $k_1 = k_2 = 0$ for simplicity and later we can recover all the k_i dependence from the covariance. For the momentum along x_3 direction, we have

$$\begin{aligned}
\widetilde{J}_{012}(k_1 = k_2 = 0, k_3) &= \frac{\mathcal{T}_{\rm D0}}{2\pi\alpha'}\left(\frac{2R}{N}\right)^2 \mathrm{tr}\left[L_3 e^{ik(2R/N)k_3 L_3}\right] \\
&= \frac{\mathcal{T}_{\rm D0}}{2\pi\alpha'}\left(\frac{2R}{N}\right)^2 \sum_{m=-(N-1)/2}^{(N-1)/2} m\, e^{imk_3(2R/N)}.
\end{aligned} \tag{4.17}$$

So from the rotational covariance we obtain

$$\widetilde{J}_{0ij}(\vec{k}) = \frac{\mathcal{T}_{\rm D0}}{2\pi\alpha'}\left(\frac{2R}{N}\right)^2 \sum_{m=-(N-1)/2}^{(N-1)/2} m\frac{\epsilon_{ijl}k_l}{k}\, e^{imk(2R/N)} . \tag{4.18}$$

Using the formula for an inverse Fourier transform

$$\frac{\epsilon_{ijl}k_l}{k}(e^{ika}-e^{-ika}) \quad \xleftarrow{\text{F.T.}} \quad -\frac{1}{2\pi a}\frac{\epsilon_{ijl}x_l}{r}\delta'(r-a) \tag{4.19}$$

and $\mathcal{T}_{\rm D2} = \mathcal{T}_{\rm D0}/(4\pi^2\alpha')$, we obtain

$$J_{0ij}(\vec{x}) = -\mathcal{T}_{\rm D2}\left(\frac{2R}{N}\right)\epsilon_{ijk}\frac{x_k}{r}\sum_{n=1}^{[N/2]}\delta'(r-R^{(n)}) . \tag{4.20}$$

This shows that locally the D2-brane charge is induced on the shells of the radii $R^{(n)}$.

Moreover, it is found that the induced D2-brane charge on a single shell is fractional. To see this, we extract only the n-th shell and compute its total charge as

$$\begin{aligned}\int d^3x\, \frac{1}{2}\epsilon_{ijk}\hat{x}_k J_{0ij}^{(n)}(\vec{x}) &= -\int 4\pi r^2 dr\, \mathcal{T}_{\rm D2}\left(\frac{2R}{N}\right)\delta'(r-R^{(n)}) \\ &= 8\pi R^{(n)}\mathcal{T}_{\rm D2}\left(\frac{2R}{N}\right) \\ &= \mathcal{T}_{\rm D2}4\pi(R^{(n)})^2\cdot\left(\frac{4R}{NR^{(n)}}\right) . \end{aligned} \tag{4.21}$$

Since the factor $4\pi(R^{(n)})^2$ is the area of the shell, we may conclude that the n-th shell has only a fraction $4/(2n-1)$ (or a fraction $2/n$) of a D2-brane for even (or odd) N.

We have encountered two strange results: D2-brane charges are induced on the shells with various radii as is the case for the D0-brane charge density, and furthermore, each shell has a fractional D2-brane charge. This strange distribution disappear in the large-N limit, as we will see in the next subsection.

4.2 Large N limit to the continuum

The results obtained in the previous subsections seem rather strange, because one might expect that N D0-branes in the configuration (4.1) form a single S^2 which is a conventional interpretation consistent with a dual D2-brane picture of a spherical brane with a magnetic field. However, in the sense of matrix regularization of membranes, at least if we take a large-N limit with fixed R, we might be able to have this dual correspondence and see the exact relation between the two. The dual D2-brane picture is based on a continuous worldvolume of D2-brane which would be reproduced by the continuum large-N limit. (Note that for the democratic duality we expect, the D-brane charge should be anyway found on a single shell even for finite N. We propose a resolution of this problem in section 7.)

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So for the present let us follow this idea of the matrix regularization and take the large-N limit carefully. We will see that although there are many shells for finite N, in the large-N limit the shells annihilate with each other and only the most outer shell remains, to give a consistent dual D2-brane picture.

First we examine the D2-brane charge density (4.20). In the large N limit, the summation over n in (4.20) can be replaced by an integration over a new variable y with the following correspondence

$$y \equiv \frac{2n}{N} R \,, \tag{4.22}$$

for odd N for simplicity. Then (4.20) becomes

$$\begin{aligned} J_{0ij}(\vec{x}) &= \lim_{N\to\infty} -\mathcal{T}_{\mathrm{D2}} \epsilon_{ijk} \frac{x_k}{r} \sum_{m>0}^{(N-1)/2} \left(\frac{2R}{N}\right) \delta' \left(r - \frac{2mR}{N}\right) \\ &= -\mathcal{T}_{\mathrm{D2}} \epsilon_{ijk} \frac{x_k}{r} \int_0^R dy\, \delta'(r-y) \\ &= \mathcal{T}_{\mathrm{D2}} \epsilon_{ijk} \frac{x_k}{r} \left[\delta(r-R) - \delta(r)\right] . \end{aligned} \tag{4.23}$$

Hence the D2-brane charge is found only on a single shell with the radius R, as mentioned. The second term $\delta(r)$ appears due to the ill-definedness of the spherical coordinates, and in fact in the total D2-brane charge it does not give any contribution due to the jacobian $4\pi r^2$ of the spherical coordinates,

$$\int 4\pi r^2 dr\, \frac{1}{2} \epsilon_{ijk} \hat{x}_k J_{0ij} = 4\pi R^2 \mathcal{T}_{\mathrm{D2}} \,. \tag{4.24}$$

Thus we have found that, in the large-N limit, the charge density formula given in [3] provides expected distribution of D2-brane charge for the fuzzy S^2 configuration of D0-branes (4.1).

Next, let us check the large-N limit of the D0-brane density (4.11). The large-N limit should give a diverging result since this N represents the number of the total D0-branes. Thus we have to divide the expression (4.11) by N so that we can see the distribution ratio of the component D0-branes.

$$\begin{aligned} \lim_{N\to\infty} \frac{1}{N} J_0(x) &= \frac{-\mathcal{T}_{\mathrm{D0}}}{4\pi R} \int_0^R dy\, \frac{1}{y} \left(\frac{1}{y} \delta(r-y) + \frac{\partial}{\partial r} \delta(r-y) \right) \\ &= \frac{-\mathcal{T}_{\mathrm{D0}}}{4\pi R} \left[\int_0^R dy\, \frac{1}{y^2} \delta(r-y) + \left[\frac{-1}{y} \delta(r-y) \right]_{y=0}^R - \int_0^R dy\, \frac{1}{y^2} \delta(r-y) \right] \\ &= \frac{\mathcal{T}_{\mathrm{D0}}}{4\pi R^2} \delta(r-R) + \frac{\mathcal{T}_{\mathrm{D0}}}{4\pi R} \lim_{y\to 0} \frac{-1}{y} \delta(r-y) \,. \end{aligned} \tag{4.25}$$

The second term is again an ill-defined quantity due to the spherical coordinates, and vanishes when we integrate over all the bulk space, as we shall see below. So only the first term is of physical interest, and this shows that in the large-N limit all the D0-branes are localized on the single sphere whose radius is R. A consistency check of the total number of D0-branes is

$$\int 4\pi r^2 dr \lim_{N\to\infty} \frac{1}{N} J_0(x) = \int 4\pi r^2 dr \left(\frac{\mathcal{T}_{\mathrm{D0}}}{4\pi R^2} \delta(r-R) + \frac{\mathcal{T}_{\mathrm{D0}}}{4\pi R} \lim_{y\to 0} \frac{-1}{y} \delta(r-y) \right) = \mathcal{T}_{\mathrm{D0}} \,.$$

We conclude that in the large-N limit the fuzzy S^2 configuration (4.1) provides a single spherical D2-brane with N D0-branes on it. The radius of this sphere is consistent with the spherical locus relation (4.3). From this observation, the strange appearance of the many shells for the finite N case can be understood as a kind of $1/N$ corrections, in a sense. It was already found in [33] that the configuration (4.1) generates correct large distance behaviour of the gravitational field up to $1/N$ corrections, which night be consistent with our results.

4.3 Another large-N limit to the fuzzy plane

We may consider varying also R as N increases, and this makes it possible to relate the fuzzy sphere with the fuzzy (noncommutative) plane studied in subsection 2.2. Basically this limit is defined as enlarging the north pole of the fuzzy sphere by taking $R \to \infty$ while keeping a finite noncommutativity on the sphere [34]. More precisely, We first define a new X_3 by a shift from the original one,

$$X_3 \equiv X_3^{\mathrm{original}} - R \mathbb{1}_N \,, \tag{4.26}$$

so that the large R limit does not bring the north pole to a spatial infinity. Then the algebra of the fuzzy sphere becomes

$$[X_1, X_2] = \frac{2R}{N} i X_3 + \frac{2iR^2}{N} \mathbb{1}_N \,, \tag{4.27}$$

while the other commutators are the same as before, (4.2). We take a large-N limit while keeping $2R^2/N \equiv \theta$ fixed, then the fuzzy sphere algebra (4.2) and (4.27) reduces to the fuzzy plane algebra (2.8).

Let us check if this limit of the results in subsections 4.1.1 and 4.1.2 reproduces those of subsection 2.2. For finite N, the D0-brane and D2-brane charges are the same as (4.11) and (4.20) except the shift in the x_3 direction, $x_3 \to x_3 + R$. Since we fix θ, the radius $R \sim \sqrt{N}$ goes to the infinity, so we may expand r as

$$r = \sqrt{(x_1)^2 + (x_2)^2 + (x_3 + R)^2} = R \left(1 + \frac{x_3}{R} + \mathcal{O}\left(\frac{1}{R^2}\right) \right) . \tag{4.28}$$

With this in mind, we may renumber the shell as $2(n'-1) = N - 2n$ ($n' = 1, 2, \ldots, N/2$) and find the D0-brane charge distribution (4.11) as

$$\begin{aligned} J_0(x) = \mathcal{T}_{\mathrm{D0}} \sum_{n'=1}^{N/2} & \frac{-1}{2\pi \left(R - \frac{2n'-1}{N} R\right)} \times \\ & \times \left(\frac{1}{R - \frac{2n'-1}{N} R} \delta\left(x_3 + \frac{2n'-1}{N} R\right) + \delta'\left(x_3 + \frac{2n'-1}{N} R\right) + \text{higher} \right) . \end{aligned} \tag{4.29}$$

Introducing a new variable $y' \equiv \frac{2n'-1}{N}R$, we find the measure of the integration is given as $dy = 2R/N = \theta/R$, and so

$$\begin{aligned}
\lim_{N\to\infty} J_0(x) &= \lim_{N\to\infty} \mathcal{T}_{\mathrm{D0}} \sum_m \frac{-1}{2\pi(R-y)} \left(\frac{1}{R-y}\delta(x_3+y) + \delta'(x_3+y)\right) \\
&= \lim_{N\to\infty} \mathcal{T}_{\mathrm{D0}} \sum_m \frac{-1}{2\pi R} \left(1+\frac{y}{R}+\cdots\right) \left(\frac{1}{R}\left(1+\frac{y}{R}+\cdots\right)\delta(x_3+y) + \delta'(x_3+y)\right) \\
&= \mathcal{T}_{\mathrm{D0}} \int_0^\infty \frac{-1}{2\pi}\frac{dy}{\theta}\delta'(x_3+y) \\
&= \frac{\mathcal{T}_{\mathrm{D0}}}{2\pi\theta}\delta(x_3)\,.
\end{aligned} \tag{4.30}$$

In the last equality we discard the delta function at the infinity. This last expression coincides with (2.11), as expected. Here, the shells disappear due to the cancellation with each other as in the previous subsection. So, even though the algebra in the large-N limit is still nontrivial, the limit makes the cancellation occur and we have finally a good agreement with the dual D2-brane picture. The results in this subsection might lead us to expect that the large N limit is necessary to have a result consistent with the dual D2-brane picture. However, we shall see in section 6 that one of the other kinds of the large-N limits is not the case.

5. Fuzzy S^4

It is straightforward to generalize the computation in the previous section to a fuzzy S^4. An explicit expression for the fuzzy S^4 constructed from D0-branes was provided in [35] as

$$X_i = \frac{R}{n}G_i^{(n)} \qquad (i = 1, 2, \ldots, 5) \tag{5.1}$$

where R is an expected radius of the sphere, and

$$G_i^{(n)} \equiv [(\Gamma_i \otimes \mathbb{1} \otimes \mathbb{1} \cdots) + (\mathbb{1} \otimes \Gamma_i \otimes \mathbb{1} \cdots) + \cdots]_{\mathrm{sym}}\,, \tag{5.2}$$

which is an n-fold symmetric tensor product of the following matrices

$$\Gamma_a = \begin{pmatrix} 0 & -i\sigma_a \\ i\sigma_a & 0 \end{pmatrix} \quad (a = 1,2,3)\,, \qquad \Gamma_4 = \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix}, \qquad \Gamma_5 = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -\mathbb{1}_2 \end{pmatrix}. \tag{5.3}$$

The rank of matrices (the total number of D0-branes) N is related to this n as

$$N = \frac{1}{6}(n+1)(n+2)(n+3)\,. \tag{5.4}$$

We have a relation analogous to (4.3) in the fuzzy S^2,

$$\sum_{i=1}^5 (X_i)^2 = R^2\left(1+\frac{4}{n}\right)\mathbb{1}_N\,, \tag{5.5}$$

which indicates that the D0-branes are on a spherical locus of the radius R corrected by $1/n$. We will see in this section that the large-N limit of the D0-brane configuration (5.2) is actually representing a bound state of n coincident spherical D4-brane and N uniformly distributed D0-branes, forming a four-sphere shell of radius R.

5.1 D-brane charges

5.1.1 D0-brane charge

Due to the rotational invariance SO(5) of the above D0-brane configuration explained in [35], which is analogous to (4.4) for the fuzzy S^2, we can bring the vector $\vec{k}$ along the x^5 axis and evaluate the D0-brane charge density, as in the case of the previous section. We obtain

$$\widetilde{J}_0(\vec{k}) = \mathcal{T}_{\mathrm{D0}}\,\mathrm{tr}\left[e^{ik(R/n)G_5^{(n)}}\right], \tag{5.6}$$

where in this section $k \equiv \sqrt{\sum_{i=1}^{5} k_i^2}$. In [35], it was shown that the number of the eigen vectors with the eigenvalue m of $G_5^{(n)}$ is $((n+2)^2 - m^2)/4$. Using this result, we obtain

$$\widetilde{J}_0(\vec{k}) = \mathcal{T}_{\mathrm{D0}} \sum_{l=0}^{n} \left(e^{ik(R/n)(2l-n)} \frac{(n+2)^2 - (2l-n)^2}{4} \right). \tag{5.7}$$

What we need for the inverse Fourier transform in 5 dimensions is that of $\delta(r-R)$ where $r \equiv \sqrt{x_1^2 + \cdots + x_5^2}$,

$$\int d^5x\, e^{ik_ix_i}\delta(r-a) = \frac{4\pi^2 a}{(ik)^3}\left[-(e^{ika} - e^{-ika}) + ika(e^{ika} + e^{ika})\right]. \tag{5.8}$$

From this equation and its derivatives, we obtain a formula of the inverse Fourier transform in 5 dimensions

$$e^{ika} + e^{-ika} \quad \overset{\text{F.T.}}{\longleftarrow} \quad \frac{1}{4\pi^2 a^2}\left[\frac{3}{a^2}\delta(r-a) + \frac{3}{a}\delta'(r-a) + \delta''(r-a)\right] \equiv D^{(4)}(r,a). \tag{5.9}$$

Using this formula, the D0-brane density is found to be

$$J_0(\vec{x}) = \begin{cases} \mathcal{T}_{\mathrm{D0}} \displaystyle\sum_{s=0}^{(n-1)/2} \frac{(n+2)^2 - (2s+1)^2}{4} D^{(4)}(r, R^{(s)}) & (\text{odd } n) \\ \mathcal{T}_{\mathrm{D0}} \dfrac{(n+2)^2}{4}\delta^5(x) + \mathcal{T}_{\mathrm{D0}} \displaystyle\sum_{s=1}^{n/2} \frac{(n+2)^2 - (2s)^2}{4} D^{(4)}(r, R^{(s)}) & (\text{even } n) \end{cases} \tag{5.10}$$

where

$$R^{(s)} \equiv \begin{cases} \dfrac{2s+1}{n} R & (\text{odd } n) \\ \dfrac{2s}{n} R & (\text{even } n) \end{cases} \tag{5.11}$$

We can see that the D0-branes form a collection of S^4 shells whose radii are given by $R^{(s)}$. The situation is similar to what we have found in the case of fuzzy S^2 in the previous section.

5.1.2 D2-brane charge

We expect that the D0-brane configuration (5.2) does not generate any D2-brane charge but gives only a D4-brane charge. The D2-brane charge $\widetilde{J}_{0ij}$ is proportional to

$$\mathrm{Str}\left([X_i, X_j]e^{ik\cdot X}\right). \tag{5.12}$$

We make again use of the SO(5) symmetry to bring it to the form

$$\mathrm{tr}\left(G_{ij}^{(n)} e^{ik_5(R/n)G_5}\right), \tag{5.13}$$

where we have defined $G_{ij}^{(n)} \equiv [G_i^{(n)}, G_j^{(n)}]/2$. When i or j is 5, the trace vanishes. For $i, j \neq 5$, we use the argument of [35] to show that this vanishes: the eigenvectors of G_5 for a given eigenvalue are split into two groups, one with positive eigenvalues of G_{ij} and the other with negative eigenvalues of it, for given i and j. The number of them are the same and so the above trace vanishes. This means that there is no D2-brane charge induced from the fuzzy S^4 configuration of D0-branes (5.2).

5.1.3 D4-brane charge

Finally let us evaluate the D4-brane charge (2.4) which is expected to be generated in a spherical shape. In the evaluation of, say, a component $\widetilde{J}_{01234}$, we encounter the following kind of traces in the symmetrization:

$$\mathrm{tr}\left(G_{12}^{(n)}\left(k_i\left(\frac{R}{n}\right)G_i^{(n)}\right)^t G_{34}^{(n)}\left(k_i\left(\frac{R}{n}\right)G_i^{(n)}\right)^{s-t}\right) \tag{5.14}$$

with a various integer t for a given integer s. We assume that after the symmetrization and the trace this is equal to[9]

$$\mathrm{tr}\left(G_{12}^{(n)} G_{34}^{(n)}\left(k_i\left(\frac{R}{n}\right)G_i^{(n)}\right)^s\right), \tag{5.15}$$

which simplifies the following calculation, since we may use the relation [35]

$$\epsilon_{ijkl5} X_i X_j X_k X_l = 8(n+2) X_5 \left(\frac{R}{n}\right)^4. \tag{5.16}$$

Using the same argument as before, from the rotational covariance under SO(5) the charge density should be of the form $\widetilde{J}_{01234} \propto k_5 f(k)$. Then we may bring $\vec{k}$ to the direction of x_5 as

$$\widetilde{J}_{01234}(\vec{k}) = \frac{\mathcal{T}_{\mathrm{D}0}}{2(2\pi\alpha')^2} 8(n+2)\left(\frac{R}{n}\right)^4 \mathrm{tr}\left(G_5^{(n)} e^{ik(R/n)G_5^{(n)}}\right)\frac{k_5}{k}. \tag{5.17}$$

The evaluation of this trace is almost the same as that for the D0-brane charge, and gives

$$\mathrm{tr}\left(G_5^{(n)} e^{ik(R/n)G_5^{(n)}}\right) = \sum_{l=0}^{n} e^{ik(R/n)(2l-n)} \frac{(n+2)^2-(2l-n)^2}{4}(2l-n). \tag{5.18}$$

Using a formula analogous to (5.8), we obtain the inverse Fourier transform for odd n for example as

$$J_{01234}(x) = \frac{\mathcal{T}_{\mathrm{D}0}}{2(2\pi\alpha')^2} 8(n+2)\left(\frac{R}{n}\right)^4 \frac{x_5}{r}\sum_{s=0}^{(n-1)/2}\frac{(n+2)^2-(2s+1)^2}{4}(2s+1)\times$$

[9] We have checked this explicitly for $n = 1$.

$$\times \frac{1}{4\pi^2 (R^{(s)})^2} \left(\frac{1}{R^{(s)}} \delta'(r - R^{(s)}) + \delta''(r - R^{(s)}) \right). \tag{5.19}$$

We have again plenty of shells of induced D4-brane charges. The location of the D4-brane shells are the same as that of D0-branes (5.10).

5.2 Large N limit to the continuum

As in the previous section of the fuzzy S^2, we shall take the large N limit and see if the above results are consistent in the continuum limit with the expected dual picture that there is a bound state of n coincident spherical D4-brane shells in which N D0-branes are bound. First let us consider the large-N limit of the D0-brane charge (5.10). In the large-N limit, we may replace the discrete sum in (5.10) by an integration over a continuous parameter

$$y \equiv \frac{2s+1}{n} R\,, \tag{5.20}$$

which is analogous to the previous (4.22), but note that in the present case the denominator is not N but n. Subsequently, the infinitesimal displacement is $\frac{2R}{n} = dy$, then the result (5.10) of the D0-brane density becomes in the large-N limit

$$\begin{aligned}
\lim_{n\to\infty} \frac{1}{N} J_0(x) &= \lim_{n\to\infty} \frac{6\mathcal{T}_{\rm D0}}{n} \sum_{s=0}^{(n-1)/2} \frac{1 - ((2s+1)/n)^2}{4} D^{(4)}(r, R^{(s)}) \\
&= \frac{3\mathcal{T}_{\rm D0}}{R} \int_0^R dy\, \frac{1}{4} \left(1 - \frac{y^2}{R^2}\right) \frac{1}{4\pi^2 y^2} \left[\frac{3}{y^2} \delta(r-y) + \frac{3}{y} \delta'(r-y) + \delta''(r-y) \right].
\end{aligned}$$

Performing partial integrations, we are finally led to the result

$$\begin{aligned}
\lim_{n\to\infty} \frac{1}{N} J_0(x) &= \frac{\mathcal{T}_{\rm D0}}{\Omega_4 R^4} \delta(r-R) + \\
&\quad + \frac{3\mathcal{T}_{\rm D0}}{16\pi^2 R^3} \lim_{y\to 0} \left[\left(\frac{R^2}{y^3} - \frac{3}{y} \right) \delta(r-y) + \left(\frac{R^2}{y^2} - 1 \right) \delta'(r-y) \right].
\end{aligned}$$

The second term is irrelevant (and in fact vanishes when integrated over the whole space), so we have confirmed that in the large-N limit all the D0-branes are on a single shell of an S^4 whose volume is $\Omega_4 R^4$ (where Ω_4 is the volume of a unit four-sphere).

Next, let us evaluate the large-N limit of the D4-brane charge (5.19). From the observation in [35], we expect that n coincident spherical D4-branes are generated. So we divide the D4-brane charge (5.19) by n and take the large-N limit. Due to the replacement (5.20), we have

$$\lim_{n\to\infty} \frac{1}{n} J_{0ijkl}(x) = \frac{\mathcal{T}_{\rm D0} R^2}{2(2\pi\alpha')^2} \epsilon_{ijklm} \frac{x_m}{r} \int_0^R dy \left(1 - \frac{y^2}{R^2}\right) \frac{1}{4\pi^2} \left(\frac{1}{y^2} \delta'(r-y) + \frac{1}{y} \delta''(r-y) \right).$$

After a partial integration, we obtain

$$\lim_{n\to\infty} \frac{1}{n} J_{0ijkl} = \mathcal{T}_{\rm D4} \epsilon_{ijklm} \frac{x_m}{r} \left[(\delta(r-R) - \delta(r)) + \frac{R^2}{2} \lim_{y\to 0} \frac{1}{y} \delta'(r-y) \right]. \tag{5.21}$$

We have used the relation $\mathcal{T}_{\rm D0} = (4\pi^2\alpha')^2\mathcal{T}_{\rm D4}$. In this result, $\delta(r)$ term and the term with $y \to 0$ are artificial due to the ill-definedness of the spherical coordinates at the origin, and actually it vanishes when integrated over the whole space as before. So we conclude that in the large-N limit, n spherical D4-branes are generated with radius R.

$$J_{0ijkl} \underset{n\to\infty}{\to} n\mathcal{T}_{\rm D4}\epsilon_{ijklm}\frac{x_m}{r}\delta(r-R)\,. \tag{5.22}$$

In fact, the total induced D4-bane charge is evaluated as

$$\lim_{n\to\infty}\frac{1}{n}\Omega_4\int r^4 dr\; J_{0ijkl}\hat{x}_m\epsilon^{ijklm}\frac{1}{4!} = \Omega_4 R^4\mathcal{T}_{\rm D4}\,. \tag{5.23}$$

6. Fuzzy cylinder

Among various fuzzy objects, one of the interesting configurations is a fuzzy cylinder [36, 37] whose shape is nontrivial. The object itself is non-compact and elongated to spacetime infinities, while the fuzzy spheres considered in the previous sections are compact objects. In this section we demonstrate similar calculations of the RR charge distributions for the fuzzy cylinder. The difference from the results of section 4 and 5 is that the representation of the fuzzy cylinder algebra is infinite dimensional, so the matrix is already in large-N, but the algebra is still nontrivial and hence not in the continuum limit.

The fuzzy cylinder is defined as a representation of the following algebra

$$[X_1, X_2] = 0\,, \qquad [X_2, X_3] = i\rho_3 X_1\,, \qquad [X_1, X_3] = -i\rho_3 X_2\,. \tag{6.1}$$

This algebra can be obtained by a certain deformation of the fuzzy S^2 algebra (4.2). This deformation will be studied in subsection 6.3. One of the explicit representations which we shall use in this paper is the following,

$$X_1 = \frac{1}{2}\rho_{\rm c}S_1\,, \qquad X_2 = \frac{1}{2}\rho_{\rm c}S_2\,, \qquad X_3 = \rho_3 S_3\,, \tag{6.2}$$

where the entries of the matrices S_i are given as

$$(S_1)_{mm'} = \delta_{m+1,m'} + \delta_{m-1,m'}\,, \tag{6.3}$$

$$(S_2)_{mm'} = i\delta_{m+1,m'} - i\delta_{m-1,m'}\,, \tag{6.4}$$

$$(S_3)_{mm'} = \left(m - \frac{1}{2}\right)\delta_{m,m'}\,. \tag{6.5}$$

Here the matrix indices m and m' run from $-\infty$ to $+\infty$, and so the representation is infinite dimensional. Note that the dimensionful parameter $\rho_{\rm c}$ does not appear in the algebra, thus a family of the representations is parameterized by a continuous $\rho_{\rm c}$. The physical meaning of this parameter is found in the following matrix equation

$$X_1^2 + X_2^2 = \rho_{\rm c}^2\mathbb{1}\,, \tag{6.6}$$

which shows that the D0-branes are expected to be on a cylindrical locus whose radius is $\rho_{\rm c}$, as in the same sense as the spherical locus relation (4.3) in the fuzzy S^2 case. The cylinder would be infinitely long since the range of the eigenvalues of X_3 covers the whole region from $-\infty$ to $+\infty$. In this section, we would like to see if these expected shape of the cylinder is actually generated in the RR charge distributions.

6.1 Isometry and continuum limit

To make sense of the D-brane charge distribution generated from matrix configurations of D0-branes, in the previous sections we have learned that a simple resolution is to take the continuum limit. This continuum limit is equivalent to the large-N limit in the previous sections, but here for the fuzzy cylinder the representation is already infinite dimensional. But this doesn't necessarily mean that we need not to take any limit further. The essence of the large-N limit in the previous sections was that it was the continuum limit in the sense of the membrane regularization — in that limit all the commutators in the relevant algebra vanish. For the fuzzy S^2 in section 4, the right hand side of the commutator (4.2) has a factor $1/N$ and thus this continuum limit is equivalent to the large-N limit with fixed R. In the present case of the fuzzy cylinder, thus we need to take the limit $\rho_3 \to 0$. For our later convenience, we write $\rho_3 = l_3/M$ and take the large M limit while l_3 is left fixed.

The matrix configuration (6.2) has a discrete translation symmetry along the x_3 direction by a shift unit ρ_3. This can be seen as follows. Consider a shift matrix U whose entry is defined as

$$U_{mm'} = \delta_{m+1,m'} \,. \tag{6.7}$$

This U is unitary. Then the translation along x_3 direction by ρ_3 can be represented by this shift operation as

$$X_i + \delta_{i,3}\mathbb{1}\rho_3 = U^{-1}X_iU \,. \tag{6.8}$$

This means that any charge density is invariant under this discrete translation. In particular, if we take the large M continuum limit, the unit of the translation goes to zero and so we may allow any amount of translation, which means that the charge densities have an isometry along x_3.

6.2 D-brane charges

Let us evaluate D-brane charge of the fuzzy cylinder configuration (6.2), in the continuum limit. The distribution before the limit is taken will be studied in the next subsection.

6.2.1 D0-brane charge

We evaluate the D0-brane charge distribution (2.1). First, consider the D0-brane density which is integrated over the x_1 and x_2 directions, by putting $k_1 = k_2 = 0$.

$$\widetilde{J}_0(k_1 = k_2 = 0, k_3) = \mathcal{T}_{\rm D0}\sum_m \left[e^{ik_3(m+1/2)\rho_3}\right]. \tag{6.9}$$

After the Fourier transformation, we obtain

$$\int dx_1 dx_2 J_0(\vec{x}) = \mathcal{T}_{\rm D0}\sum_m \delta\left(x_3 - \left(m+\frac{1}{2}\right)\rho_3\right). \tag{6.10}$$

If we take the large M limit, we have a continuous uniform distribution of D0-branes,

$$\lim_{M\to\infty}\frac{1}{M}\int dx_1 dx_2 J_0(\vec{x}) = \frac{\mathcal{T}_{\rm D0}}{l_3}\,. \tag{6.11}$$

The density per a unit length along x_3 is M/l_3.

To see the full spatial dependence of the charge distribution in the large M limit, we may put $k_3 = 0$ since we already know that the distribution is translationally symmetric along x_3 in this limit. To compute the density

$$\widetilde{J}_0(k_1, k_2, k_3 = 0) = \mathcal{T}_{\rm D0}\,{\rm tr}\left[e^{ik_1X_1 + ik_2X_2}\right], \tag{6.12}$$

we note that this expression is invariant under the rotation around the axis x_3. So we may put $k_2 = 0$ and recover its $\vec{k}$ dependence by just replacing k_1 by $k_\rho \equiv \sqrt{k_1^2 + k_2^2}$. Then

$$\begin{aligned}\widetilde{J}_0(k_1, k_2, k_3 = 0) &= \mathcal{T}_{\rm D0}\,{\rm tr}\left[e^{ik_\rho(\rho_{\rm c}/2)S_1}\right] \\ &= \mathcal{T}_{\rm D0}({\rm tr}\,\mathbb{1}) \sum_{n\,:\,{\rm even}} \frac{1}{n!}\left(\frac{ik_\rho\rho_{\rm c}}{2}\right)^n {}_nC_{n/2} \\ &= \mathcal{T}_{\rm D0}({\rm tr}\,\mathbb{1})\,B_0(k_\rho\rho_{\rm c}),\end{aligned} \tag{6.13}$$

where $B_0(\tau)$ is a Bessel function. Using an integral expression of the Bessel function,

$$B_0(\tau) = \frac{2}{\pi}\int_0^1 \frac{\cos(\tau s)}{\sqrt{1-s^2}}ds = \frac{1}{\pi}\int_0^\pi d\theta\, e^{i\tau\cos\theta}, \tag{6.14}$$

and the Fourier transform of the delta function of $\rho = \sqrt{x_1^2 + x_2^2}$,

$$\int dx_1 dx_2\, e^{ik_1x_1 + ik_2x_2}\delta(\rho - a) = 2a\int_0^\pi d\theta\, e^{ik_\rho a\cos\theta}, \tag{6.15}$$

we obtain the inverse Fourier transform of the density as

$$\int dx_3 J_0(\vec{x}) = \mathcal{T}_{\rm D0}({\rm tr}\,\mathbb{1})\frac{1}{2\pi\rho_{\rm c}}\delta(\rho - \rho_{\rm c}). \tag{6.16}$$

Using the fact that the integration over x_3 is reduced to a multiplication of the factor ${\rm tr}\,\mathbb{1}\rho_3$ in the large M limit, we may write the above expression as a local one,

$$\lim_{M\to\infty}\frac{1}{M}J_0(\vec{x}) = \mathcal{T}_{\rm D0}\frac{1}{2\pi\rho_{\rm c}l_3}\delta(\rho - \rho_{\rm c}). \tag{6.17}$$

This expression shows that the D0-branes are distributed on a cylinder with radius $\rho_{\rm c}$. The cylinder is extended along x_3 in the three dimensional space.

6.2.2 D2-brane charge

Next, let us compute the D2-brane charge. The orientation of the generated D2-brane is along the x_3 axis, because the following component of the D2-brane charge expression

$${\rm tr}\left[[X_1, X_2]e^{ikX}\right] \tag{6.18}$$

vanishes due to the algebra of the fuzzy cylinder, (6.1).

Let us compute the 023 component of the D2-brane charge

$$\widetilde{J}_{023}(\vec{k}) = \frac{\mathcal{T}_{\rm D0}}{2\pi\alpha'}\,{\rm tr}\left[-i[X_2, X_3]e^{ik_iX_i}\right] = \frac{\mathcal{T}_{\rm D0}\rho_3}{2\pi\alpha'}\,{\rm tr}\left[X_1 e^{ik_iX_i}\right]. \tag{6.19}$$

Again we put $k_3 = 0$ to evaluate the charge density. Due to the fact that the expression is covariant with respect to the rotation around the x_3 axis, the evaluation of the trace should give a function of the form $k_1 f(k_\rho)$. Hence we may put $k_2 = 0$ and later will recover the k_i dependence. For $k_2 = 0$,

$$\begin{aligned}
\mathrm{tr}\left[X_1 e^{ik_1X_1}\right] &= \frac{\rho_\mathrm{c}}{2}\,\mathrm{tr}\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{ik_1\rho_\mathrm{c}}{2}\right)^n (S_1)^{n+1} \\
&= \frac{\rho_\mathrm{c}}{2}(\mathrm{tr}\,\mathbb{1})\sum_{n\,:\,\mathrm{odd}>0}^{\infty}\frac{1}{n!}\left(\frac{ik_1\rho_\mathrm{c}}{2}\right)^n {}_{n+1}C_{(n+1)/2} \\
&= (\mathrm{tr}\,\mathbb{1})\frac{1}{ik_1}\sum_{j=1}^{\infty}\frac{2j}{(j!)^2}\left(-\frac{k_1^2\rho_\mathrm{c}^2}{4}\right)^j \\
&= (\mathrm{tr}\,\mathbb{1})\frac{1}{i}\frac{\partial}{\partial k_1}B_0(k_1\rho_\mathrm{c})\,.
\end{aligned} \tag{6.20}$$

So, recovering the covariance, we obtain

$$\widetilde{J}_{023}(k_1,k_2,k_3=0) = \frac{\mathcal{T}_{\mathrm{D}0}\rho_3}{2\pi i\alpha'}\frac{\partial}{\partial k_1}J_0(k_\rho\rho_\mathrm{c})\,. \tag{6.21}$$

The Fourier transform is given by

$$\int dx_3\, J_{023} = \frac{\mathcal{T}_{\mathrm{D}0}\rho_3}{2\pi\alpha'}(\mathrm{tr}\,\mathbb{1})\frac{x_1}{2\pi\rho_\mathrm{c}}\delta(r-\rho_\mathrm{c})\,. \tag{6.22}$$

This shows that the local D2-brane charge is generated on the cylinder. In the large M limit, the D2-brane charge distribution obtains the isometry along x_3, thus we are led to the following local expression

$$\lim_{M\to\infty} J_{023} = \frac{\mathcal{T}_{\mathrm{D}0}}{2\pi\alpha'}\frac{x_1}{2\pi\rho_\mathrm{c}}\delta(r-\rho_\mathrm{c})\,. \tag{6.23}$$

This coincides with what we have expected. The single D2-brane forms a complete continuous cylinder with the radius ρ_c oriented along x_3. In fact, the total D2-brane charge can be computed as

$$\int dx_3 \int 2\pi\rho d\rho\,(J_{023}\hat{x}_1 - J_{013}\hat{x}_2) = \frac{\mathcal{T}_{\mathrm{D}0}}{4\pi^2\alpha'}\cdot(\mathrm{tr}\,\mathbb{1})\rho_3\cdot 2\pi\rho_\mathrm{c}\,. \tag{6.24}$$

Since $(\mathrm{tr}\,\mathbb{1})\rho_3$ is the length of the cylinder (which is infinitely long), we can rewrite this as $\mathcal{T}_{\mathrm{D}2}$ times the total area of the cylinder.

6.3 From fuzzy S^2 to fuzzy cylinder

In the previous subsection we made full use of the isometry in the continuum limit, but our concern is distribution of the charges before the continuum limit is taken. For this purpose we can use the fact that the fuzzy cylinder can be obtained as a special large-N limit of an ellipsoidal deformation of the fuzzy S^2. In this subsection we consider this deformation to obtain the charge distribution of the fuzzy cylinder for $M = 1$, and see how the results obtained in this section are consistent with the results found in section 4.

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We generalize the fuzzy sphere (4.1) to a fuzzy membrane of an ellipsoidal shape, as

$$X_i = \frac{2\alpha_i R}{N} L_i \tag{6.25}$$

in which we introduced asymmetry factors α_i $(i = 1, 2, 3)$ for each of the three spatial directions. With this deformation the algebra (4.2) is modified accordingly to

$$[X_1, X_2] = \frac{2R}{N}\frac{\alpha_1\alpha_2}{\alpha_3} iX_3\,, \qquad [X_2, X_3] = \frac{2R}{N}\frac{\alpha_2\alpha_3}{\alpha_1} iX_1\,, \qquad [X_3, X_1] = \frac{2R}{N}\frac{\alpha_3\alpha_1}{\alpha_2} iX_2\,. \tag{6.26}$$

To obtain the algebra of the fuzzy cylinder (6.1) from that of the deformed fuzzy sphere (6.26), we take the large-N limit with fixed α_1 and α_2 as

$$\alpha_3 = \frac{N}{2} \to \infty\,, \qquad R = \rho_3\,. \tag{6.27}$$

In this limit, the spherical locus relation of the fuzzy sphere (4.3) is reduced to that of the fuzzy cylinder (6.6), if we take

$$\alpha_1 = \alpha_2 = \frac{\rho_c}{\rho_3}\,. \tag{6.28}$$

Note that this large-N limit does not make the algebra trivial. To have a continuum limit we should further take the large M limit with $l_3 \equiv \rho_3 M$ fixed.

The explicit expression of the N dimensional representation of SU(2) is (4.6) and

$$(L_+)_{mm'} \equiv (L_1 + iL_2)_{mm'} = \sqrt{(m'-1)(N-m'+1)}\,\delta_{m+1,m'}\,, \tag{6.29}$$

$$(L_-)_{mm'} \equiv (L_1 - iL_2)_{mm'} = \sqrt{(N-m')m'}\,\delta_{m-1,m'}\,. \tag{6.30}$$

In the large-N limit (6.27)(6.28), we concentrate on a part of the entries with $m, m' \sim N/2$ (center parts of the matrices), then we can reproduce the representation (6.2) of the fuzzy cylinder algebra.

The D0-brane charge density before the continuum limit is taken can be evaluated precisely in the same way as in section 4. The only difference is that the three spatial directions have different scale factors α_i. The D0-brane charge density is given as

$$\widetilde{J}_0(k_i) = \mathcal{T}_{\rm D0}\, \mathrm{tr}\left[e^{ik'_i(2R/N)L_i}\right] \tag{6.31}$$

in which $k'_i \equiv \alpha_i k_i$. Then from the rotational invariance in this scaled momentum space, we obtain

$$\widetilde{J}_0(k_i) = \mathcal{T}_{\rm D0} \sum_{m=-(N-1)/2}^{(N-1)/2} \exp\left[ik' m \frac{2R}{N}\right]. \tag{6.32}$$

We have defined $k' \equiv |\vec{k'}| = \sqrt{\alpha_1^2 k_1^2 + \alpha_2^2 k_2^2 + \alpha_3^2 k_3^2}$. The inverse Fourier transform of this for even N for example is obtained as

$$J_0(\vec{x}) = \frac{1}{\alpha_1\alpha_2\alpha_3} \mathcal{T}_{\rm D0} \sum_{n=1}^{N/2} D(r', R^{(n)}) \tag{6.33}$$

where $R^{(n)}$ is defined in (4.12) and

$$r' \equiv \sqrt{\left(\frac{x_1}{\alpha_1}\right)^2 + \left(\frac{x_2}{\alpha_2}\right)^2 + \left(\frac{x_3}{\alpha_3}\right)^2} = \sqrt{\frac{(\rho_3)^2}{(\rho_{\rm c})^2}\left((x_1)^2 + (x_2)^2\right) + \frac{4}{N^2}(x_3)^2}\,. \tag{6.34}$$

This shows that the shells are deformed ellipsoidally with the scale factors α_i, for finite N.

Let us take the large-N limit to obtain the fuzzy cylinder. We consider first the case $\rho^2 = (x_1)^2 + (x_2)^2 \neq 0$. The large-$N$ limit makes the x_3 dependence in r' vanish, then $r' = \rho\rho_3/\rho_{\rm c}$. In the large-$N$ limit the summation over the shells can be replaced by an integration, and a computation gives

$$J_0^{(\rho\neq 0)}(x) = \frac{\mathcal{T}_{\rm D0}}{2\pi}\frac{\rho_3}{\rho_{\rm c}}\left[\delta(r'-\rho_3) - \lim_{y\to 0}\frac{\rho_3}{y}\delta(r'-y)\right]. \tag{6.35}$$

In two dimensions the second term is nontrivial (while in three dimensions in section 4 the corresponding term is an artifact of the spherical coordinates) and actually rewritten as

$$J_0^{(\rho\neq 0)}(x) = \frac{\mathcal{T}_{\rm D0}}{2\pi\rho_3\rho_{\rm c}}\delta(\rho-\rho_{\rm c}) - \frac{\mathcal{T}_{\rm D0}}{\rho_3}\delta(x_1)\delta(x_2)\,. \tag{6.36}$$

The argument of the second term should be understood as a limit of approaching to the origin. Interestingly, the integration of $J_0^{(\rho\neq 0)}(x)$ over x_1 and x_2 vanishes,

$$\int dx_1 dx_2\, J_0^{(\rho\neq 0)}(x) = 0\,. \tag{6.37}$$

The first term of (6.36) gives a continuous cylindrical distribution of the D0-branes while the second term is anti-D0-branes smeared on the x_3 axis.

On the other hand, when $x_1 = x_2 = 0$, the situation is very different. Since $r' = 2|x_3|/N$, the summation over the shells cannot be replaced by the integration. To look closely at the behaviour around the polar origin $x_1 = x_2 = 0$, we consider the case of $x_1 \sim x_2 \sim \mathcal{O}(1/N)$, and introduce a new $\mathcal{O}(1)$ variable $\tilde{\rho}^2 \equiv (N^2/4)(\rho_3/\rho_{\rm c})^2[(x_1)^2 + (x_2)^2]$. Then, r' becomes of order $\mathcal{O}(1/N)$,

$$r' = \left(\frac{2}{N}\right)\tilde{r}'\,, \qquad \tilde{r}' \equiv \sqrt{\tilde{\rho}^2 + (x_3)^2}\,. \tag{6.38}$$

After a straightforward calculation one finds

$$J_0^{(\rho\sim 0)}(x) = \frac{\mathcal{T}_{\rm D0}}{2\pi(\rho_{\rm c})^2}\sum_n \frac{-N^2}{(2n-1)^2}\left[\delta\left(\tilde{r}' - \left(n-\frac{1}{2}\right)\rho_3\right) + \rho_3\left(n-\frac{1}{2}\right)\delta'\left(\tilde{r}' - \left(n-\frac{1}{2}\right)\rho_3\right)\right].$$

It is obvious that this term gives a diverging contribution at $x_3 \sim (n-1/2)\rho_3$ and $x_1 \sim x_2 \sim 0$, so we compute an integration of this over a small region in the x_1-x_2 plane around the origin $x_1 = x_2 = 0$. Using the calculation (4.16), we obtain

$$\int_{x_1,x_2\sim 0} dx_1 dx_2\, J_0^{(\rho\sim 0)}(x) = \sum_n \mathcal{T}_{\rm D0}\,\delta\left(x_3 - \left(n-\frac{1}{2}\right)\rho_3\right). \tag{6.39}$$

Therefore we may conclude that the local expression of this is precisely a delta function in the x_1-x_2 space, as

$$J_0^{(\rho\sim 0)}(x_1\sim 0, x_2\sim 0, x_3) = \mathcal{T}_{\rm D0}\sum_n \delta(x_1)\delta(x_2)\,\delta\left(x_3 - \left(n-\frac{1}{2}\right)\rho_3\right), \tag{6.40}$$

in the large-N limit. This result $J_0^{(\rho\sim 0)}(x)$ shows the presence of a sequence of un-smeared D0-branes on the x_3 axis.

Adding the two contributions we obtain the full D0-brane distribution, $J_0 = J_0^{(\rho\neq 0)} + J_0^{(\rho\sim 0)}$. This full expression is consistent with the results in the previous subsection, (6.10) and (6.16), when integrated over a part of the spatial directions. Of course, in the continuum limit $M\to\infty$, it is easy to show that the result coincides with (6.17):

$$\lim_{N,M\to\infty}\frac{1}{M}\frac{1}{\alpha_1\alpha_2\alpha_3}\mathcal{T}_{\rm D0}\sum_{n=1}^{N/2} D(r', R^{(n)}) = \mathcal{T}_{\rm D0}\frac{1}{2\pi\rho_{\rm c} l_3}\delta(\rho-\rho_{\rm c})\,. \tag{6.41}$$

In the continuum limit, the second term in $J^{(\rho\neq 0)}$ is cancelled completely by $J^{(\rho\sim 0)}$, and only the first term in $J^{(\rho\neq 0)}$ is left, which is the continuous cylinder.

We conclude that also for the fuzzy sphere, although the representation is infinite dimensional, the D-brane charge distribution is very strange and does not agree with the dual D2-brane picture. Without taking the continuum limit, we find smeared anti-D0-brane and scattered D0-branes on the x_3 axis.

The local expression of the D2-brane charge (6.23) is reproduced in the same manner, in the large N, M limit. One finds that the fuzzy cylinder without taking the continuum limit has the same D2-brane charge distribution as that of the continuum limit, interestingly.

6.4 Fuzzy supertube

So far in sections 4, 5 and 6, we have concentrated on the determination of the RR charge distribution from a given fuzzy configuration of D0-branes. But in actual situations, one has to solve equations of motion of the effective D0-brane action to obtain those fuzzy configurations as classical solutions. Various fuzzy solutions are known to emerge. For example, the dimension N fuzzy S^2 solution appears in the background constant RR four-form field strength [5], and this static fuzzy solution is expected to have a "dual" description — a spherical D2-brane on which there are N units of magnetic field to represent the bound N D0-branes. In this subsection, we present a more intuitive way to realize the fuzzy cylinder which we considered in the previous subsections, as a classical solution. It turns out that the resultant configuration is a fuzzy supertube [38].

To obtain extended fuzzy objects in the bulk spacetime as static classical solutions, one has to introduce some mechanism which keeps the extended state stable. In the case of the above fuzzy S^2, it is due to the external background field strength. One of the other ways to obtain the fuzzy spheres is via the Nahm construction of the monopole, in which the fuzzy S^2 appears as a part of a fuzzy funnel which is a fuzzy expansion of D-strings and represents the D-strings ending on D3-branes [39]. In this case the stability is achieved by introducing nontrivial boundary conditions at the end of the worldvolume of the D-strings.

A naive way to stabilize extended structures without turning on any background is to introduce a mechanical angular momentum. This makes it possible to have fuzzy structure with axial symmetry, as solutions of the equations of motion for D0-branes:

$$-\ddot{X}_i + \frac{1}{(2\pi\alpha')^2}[X_j,[X_i,X_j]] = 0\,, \qquad [X_i,\dot{X}_i] = 0\,. \tag{6.42}$$

The second equation is the Gauss low, the dot denotes the time derivative, and we took the gauge $A_0 = 0$. A solution with the angular momentum in the x_1-x_2 plane is

$$\begin{aligned} X_1 &= X_1^{(0)} \cos\omega t + X_2^{(0)} \sin\omega t\,, \\ X_2 &= -X_1^{(0)} \sin\omega t + X_2^{(0)} \cos\omega t\,, \\ X_3 &= X_3^{(0)}\,, \end{aligned} \tag{6.43}$$

where $X_i^{(0)}$ are the matrices satisfying the fuzzy cylinder algebra (6.1). In the following, we shall use the explicit representation of the fuzzy cylinder, (6.2). Immediately we find that to satisfy the equations of motion we have to take

$$\omega = \frac{\rho_3}{2\pi\alpha'}\,. \tag{6.44}$$

It is interesting that the radius of the cylinder, ρ_c, is not related to the angular velocity ω. This reminds us of a supertube [38] which has a degree of freedom to deform arbitrarily the shape of its cross section. The arbitrariness of ρ_c is related to this degrees of freedom of the supertube. As seen below, actually our rotating cylinder has a supersymmetry. The solution (6.43) is also a solution of the following BPS equations of the D0-brane system [36],

$$\begin{aligned} [X_1,X_2] &= 0\,, & D_0X_3 &= 0\,, \\ D_0X_1 \pm i[X_3,X_1] &= 0\,, & D_0X_2 \pm i[X_3,X_2] &= 0\,. \end{aligned} \tag{6.45}$$

The authors of [36] took a different gauge $X_3 = A_0$ to solve these BPS equations. Our solution is just in a gauge different from theirs, and has a clear mechanical origin.

6.4.1 RR densities

Let us evaluate the charges generated by the solution (6.43), to see if this configuration of the rotating D0-branes is actually a supertube consisting of a cylindrical D2-brane. We consider only the continuum limit of the charge distribution, since without this we would get strange charge distribution as found in subsection 6.3.

First, we evaluate densities of the RR charges. In evaluation, we can use the results of the previous subsections. Since the charge density formulas of the D0-branes (2.1) and D2-branes (2.3) do not include any time derivative, the results for the rotating solution (6.43) can be obtained by simply replacing k_1 by $(k_1\cos\omega t - k_2\sin\omega t)$ and k_2 by $(k_1\sin\omega t + k_2\cos\omega t)$ in (6.13) and (6.21). However, this replacement does not change the magnitude of the momentum $k_\rho = \sqrt{k_1^2+k_2^2}$. Hence we conclude that the D2-brane and D0-brane charge densities in the rotating case are the same as those of the previous subsections. This shows that in the large N, M limit the RR charges are generated at the precise location needed for the identification with the D2-brane supertube with the radius ρ_c.

The D0-brane motion is expected to be along the tube on which those D0-branes are smeared, so we can never see the motion of the D0-brane by looking just at the time component of the D0-brane density $J_0(x)$. However, even in this kind of situation we can extract the information of the velocity of the D0-branes by looking at the other components of the D0-brane density,

$$\widetilde{J}_i(k) \equiv \mathcal{T}_{\rm D0}\,{\rm Str}\left[\dot{X}_i e^{ik_iX_i}\right]. \tag{6.46}$$

The reason is obvious. The coupling of a single D0-brane to its relevant RR gauge field is

$$\frac{S_{\rm RR}}{\mathcal{T}_{\rm D0}} = \int C^\mu dX_\mu = \int C^\mu(X(\tau))\partial_\tau X_\mu(\tau)d\tau\,. \tag{6.47}$$

If we take the static gauge $X_0=\tau$ and turn on a velocity for the D0-brane as $\partial_\tau X_1 = v_1$, then the above coupling is written as

$$\frac{S_{\rm RR}}{\mathcal{T}_{\rm D0}} = \int C_0(X)d\tau + \int C_1(X)v_1 d\tau\,. \tag{6.48}$$

So, once all the components of the D0-brane density $J_\mu(x) \equiv \delta S_{\rm RR}/\delta C_\mu$ are known, one can compute the velocity of the D0-branes by the formula $v_i = J_i(x)/J_0(x)$.

The evaluation of (6.46) is almost the same as the evaluation of the D2-brane density, and the final result is (we put $k_3=0$ as before)

$$\int dx_3\, J_1(x) \;=\; \omega x_2 \mathcal{T}_{\rm D0}({\rm tr}\,\mathbb{1})\frac{1}{2\pi\rho_{\rm c}}\delta(\rho-\rho_{\rm c})\,, \tag{6.49}$$

$$\int dx_3\, J_2(x) \;=\; -\omega x_1 \mathcal{T}_{\rm D0}({\rm tr}\,\mathbb{1})\frac{1}{2\pi\rho_{\rm c}}\delta(\rho-\rho_{\rm c})\,, \tag{6.50}$$

$$J_3(x) \;=\; 0\,. \tag{6.51}$$

So the velocity is given as

$$\vec{v} = \left(\frac{J_1}{J_0},\frac{J_2}{J_0},\frac{J_3}{J_0}\right) = (\omega x_2, -\omega x_1, 0)\,. \tag{6.52}$$

This shows that the D0-branes forming the cylinder are rotating on the surface with the angular velocity ω, as expected. This result might hold even for finite M.

6.4.2 NSNS density and supersymmetry

Supertubes have F-string charge as well as the D-brane charges, and from the dual picture we expect that this F-string charge is generated on the cylinder along the direction of the x_3 axis. The F-string density formula (2.7) is evaluated in our case as

$$\begin{aligned}\widetilde{I}_{03}(\vec{k}) \;&=\; \frac{\mathcal{T}_{\rm D0}}{2\pi\alpha'}\,{\rm Str}\left[i[X_3,X_1]\dot{X}_1 e^{ik_iX_i} + i[X_3,X_2]\dot{X}_2 e^{ik_iX_i}\right] \\ &=\; \frac{\mathcal{T}_{\rm D0}}{2\pi\alpha'}\omega\rho_3\,{\rm Str}\left[(X_1X_1+X_2X_2)e^{ik_iX_i}\right].\end{aligned} \tag{6.53}$$

As before, we may put $k_3=0$, then due to the commutation relation $[X_1,X_2]=0$ we may replace Str by the normal trace. Using the relation (6.6), we obtain

$$\widetilde{I}_{03}(k_1,k_2,k_3=0)=\frac{\omega\rho_3\rho_{\rm c}^2}{2\pi\alpha'}\widetilde{J}_0(k_1,k_2,k_3=0)\,. \tag{6.54}$$

Hence after the inverse Fourier transformation,

$$\int dx_3\, I_{03}(x)=\frac{\mathcal{T}_{\rm D0}}{2\pi\alpha'}\omega\rho_3\rho_{\rm c}^2\,({\rm tr}\,\mathbb{1})\,\frac{1}{2\pi\rho_{\rm c}}\delta(\rho-\rho_{\rm c})\,. \tag{6.55}$$

It can be shown that the other components of the F-string charge vanish due to the fuzzy cylinder algebra (6.1), therefore we conclude that the fundamental strings oriented in the x_3 direction are generated uniformly on the cylinder with the radius ρ_c. Noting that $\rho_3({\rm tr}\,\mathbb{1})$ is the length of the cylinder in the x_3 direction, the above result indicates a local expression in the large M limit

$$I_{03}(x)=\frac{\mathcal{T}_{\rm D0}}{2\pi\alpha'}\omega\rho_{\rm c}^2\frac{1}{2\pi\rho_{\rm c}}\delta(\rho-\rho_{\rm c})\,. \tag{6.56}$$

Let us see that this resultant amount of the fundamental string charge is precisely reproducing the corresponding quantities calculated in the dual D2-brane description of the supertube. The supersymmetry condition of cylindrically symmetric D2 supertubes is simply written as

$$|\mathbf{D}||\mathbf{B}|=1\,, \tag{6.57}$$

where $\mathbf{D}$ is an electric induction on the D2-brane while $\mathbf{B}$ is a magnetic field. As is well known, $\mathbf{B}$ is related to the D0-brane charge density on the D2-brane, as

$$\mathcal{T}_{\rm D2}|\mathbf{B}|\delta(\rho-\rho_{\rm c})=J_0(x)\,, \tag{6.58}$$

while $|\mathbf{D}|$ is related to the F-string charge as

$$\mathcal{T}_{\rm D2}|\mathbf{D}|\delta(\rho-\rho_{\rm c})=I_{03}(x)\,. \tag{6.59}$$

We use our results of the charge density (6.17) and (6.56), to derive the dictionary

$$|\mathbf{B}|=\frac{1}{\rho_3}\frac{1}{2\pi\rho_{\rm c}}\frac{\mathcal{T}_{\rm D0}}{\mathcal{T}_{\rm D2}}\,,\qquad |\mathbf{D}|=\frac{\omega\rho_{\rm c}^2}{2\pi\alpha'}\frac{1}{2\pi\rho_{\rm c}}\frac{\mathcal{T}_{\rm D0}}{\mathcal{T}_{\rm D2}}\,. \tag{6.60}$$

Substituting (6.44) coming from the equations of motion into these equations, we can show that these quantities actually satisfy the supersymmetry condition (6.57). This provides a proof that our rotating D0-branes in the large M limit form a single D2-supertube.

7. Corrections to the density formulas and the duality

7.1 Discussions for the duality with finite N

In section 4, 5 and 6, we have seen that the RR charge density of *the continuum limit* of the fuzzy objects coincide with the dual picture of the spherical (or cylindrical) D2-brane (or D4-brane) with smeared D0-branes bound on it. This is basically what has been expected in the sense of the matrix regularization. To see this in more detail, we recall the fact that in the continuum limit of the matrix regularization we may replace the trace of the matrices by the two dimensional integration over the spatial worldvolume of the membrane (parameterized by σ_i) [40],

$$\frac{\int d^2\sigma M^{\rm cont}(\sigma_i)}{\int d^2\sigma} \sim \frac{1}{N}\,{\rm tr}\, M\,, \tag{7.1}$$

where $M^{\rm cont}(\sigma)$ is the membrane configuration corresponding to the large-N limit of the matrices M. This results in the evaluation of $J_0(x)$ in our case as

$$\int d^dk\, e^{-ik_ix_i}\frac{1}{N}\,{\rm tr}\left[e^{ik_iX_i}\right] \sim \frac{1}{\int d^2\sigma}\int d^dk\, e^{-ik_ix_i}\int d^2\sigma\, e^{ik_iX_i^{\rm cont}(\sigma)} \sim \delta(x_i - X_i^{\rm cont}(\sigma))\,,$$

which coincides with our results obtained in section 4, 5 and 6. In this sense, we can say that what we have computed in this paper is a precise version of this "$\sim$" relation by explicit evaluations of the trace in the matrix configurations, for respective supergravity couplings. This "$\sim$" makes sense only in the continuum limit where the algebra becomes trivial.

However, before taking the continuum limit, we encounter strange distribution of the RR charges — the collection of many spherical shells. Does this finite N (or finite M) configuration make sense for the democratic duality with finite N?

In a certain case, it is possible to make sense of the charge distributions while the fuzziness is kept. The example is the fuzzy (noncommutative) plane (2.8). Though the representation is infinite dimensional, the manifold itself is non-compact and thus the fuzziness still remains, as seen in the fact that the D0-brane density in a unit area of the plane is finite ($= 1/2\pi\theta$). This is reflected in the fact that, although the dimension of the representation is infinite, the algebra itself is still nontrivial. In this sense we haven't took the continuum limit for this fuzzy plane. There is a dual D2-brane description of this RR charge distribution. It is a flat bound state of a D2-brane and smeared D0-branes, which reproduces the charge distributions (2.11) and (2.10). The resultant brane configuration has a worldsheet description as a conformally invariant boundary of the worldsheet with mixed boundary conditions. From this example, we might naively expect that even for finite N (or more precisely, before taking the continuum limit, i.e. when the algebra is nontrivial), the D0-brane picture should agree with the dual D2-brane (D4-brane) picture, giving the same distribution of the RR charges. However, in the example of the fuzzy S^2, we found in section 4 that this is obviously violated. First, there are shells, and second, the shells have fractional D2-brane charges. Why in this case of the fuzzy spheres do the charge distributions disagree with the dual D2-brane picture?

We discuss possible resolutions to this interesting puzzle. Since there is no known worldsheet description of fractional D-branes without any orbifolding, we believe that the results presented in this paper should be corrected for finite N, somehow. The finite N corrections would come from the commutators of X_i's, thus the possible corrections to the RR charge density formulas are of this commutator type. There is a naive reason why we may expect commutator corrections: T-dual of the commutators are covariant derivatives, and we may expect the derivative corrections since the density formulas have been derived at the low energy. So a priori there seems to be no reason why we may discard the commutator corrections in the charge density formulas. This includes the possibility of changing the ordering prescription in the density formulas.

In some situations with supersymmetries, one might discard the derivative corrections since they are cancelled with each other (see [41]). On the other hand, we may embed the fuzzy S^2 into a fuzzy funnel [39] and in this way the fuzzy spheres we have treated can be a supersymmetric configuration. Therefore we might argue that the fuzzy sphere may receive no correction. However, precisely speaking, there is no proof that all the higher derivative corrections vanish by the preserved supersymmetries, so in a strict sense we cannot rely on the supersymmetry.

One of the other possibilities to resolve the discrepancy is concerning whether the matrix configuration is on-shell or not. Since the above belief on the duality of the pictures is based on a unified description a la boundary of string worldsheet, and so we might naively guess that the duality of the pictures might be present only for a conformally invariant boundary conditions. This means that, to show the democratic duality, the D0-brane configurations (and also the dual D2-brane configurations) should be on-shell and satisfy their equations of motion. (If the configuration of X_i is off-shell, the corresponding boundary state (2.2) may contain divergence [42].) In our situation, it is known that the static fuzzy sphere itself is not a solution of the equations of motion for the D0-branes in a flat background, and to make it a solution one has to introduce a constant RR 4-form field strength background [5], or a near horizon geometry of an NS5-brane, for example. At the core of the NS5-brane, the spacetime is not flat but curved and in fact becomes S^3. The fuzzy S^2 appears as a conjugacy class of an SU(2) ($\sim S^3$) group manifold. If we assume that the density formula (2.1) persists in this curved background, naively we have to perform the Fourier transformation on S^3 (and not R^3 which led us to the collections of shells). So it might be possible to make the shells disappear due to the different metric in the momentum space.[10] However, note that if we embed the fuzzy S^2 into the fuzzy funnel as discussed above, it becomes on-shell in a flat spacetime and this kind of argument doesn't apply.

One way to see indirectly the geometry of given matrices is to calculate the fluctuation spectrum around the matrix configuration in D0-brane effective field theory and match it with a spectrum of a string with worldsheet boundary conditions corresponding to the expected brane configuration. If there really are the collection of the shells, an interaction of each shell with a probe D0-brane would give tachyonic fluctuations. The fluctuation spectrum of the fuzzy sphere solution with a separated single D0-brane, corresponding to the present situation, has been computed in [44] and the conclusion was that there is a single spherical D2-brane, no such collection of the shells. However, one should note that the spectrum calculation was performed with the lowest order action without any higher derivative terms and for a large N. So this result of the fluctuation analysis can receive nontrivial commutator corrections.

[10]See [43] for a related discussion.

In sum, various arguments above suggest that in the charge density formulas there should be $1/N$ corrections which make the strange shells vanish to give a single spherical D-brane.[11] Those might be due to the possible commutator (higher derivative or ordering) corrections.[12]

7.2 Corrections to the formulas

In the following, we shall derive the first nontrivial commutator correction to the D0-brane charge density formula (2.1), assuming that the corrected density formula gives only a single shell of the D0-brane distribution for the fuzzy S^2 and the fuzzy S^4 with finite N.

Let us consider the multipole expansion of the D0-brane charge density. When X_i's can be diagonalized simultaneously, the formula (2.1) gives a correct result in which the D0-branes are distributed with the delta-function support on the points specified by the eigenvalues of the X_i. Hence the corrections should be of the commutator form, and the first nontrivial correction in expansion in terms of k should be

$$\frac{\widetilde{J}_0(\vec{k})}{\mathcal{T}_{\rm D0}} = \left(\mathrm{tr}\left[e^{ik_iX_i}\right] + ck^4\ \mathrm{tr}\left[[X_i,X_j]^2\right] + \widetilde{c}k^2\ \mathrm{tr}\left[[X_i,k_lX_l][X_i,k_mX_m]\right] + \mathcal{O}(k^6)\right), \quad (7.2)$$

where c and $\widetilde{c}$ are constants which we will determine from the assumption mentioned above.

First we study the fuzzy S^2. If the resulting D0-brane distribution is a single sphere with its radius a, then (4.8) shows that its momentum representation is

$$\begin{aligned}\frac{\widetilde{J}_0(\vec{k})}{\mathcal{T}_{\rm D0}} &= \mathcal{N}\frac{-2\pi ia}{k}\left(e^{ika} - e^{-ika}\right) \\ &= 4\pi a^2\mathcal{N}\left[1 - \frac{1}{3!}k^2a^2 + \frac{1}{5!}k^4a^4 + \mathcal{O}(k^6)\right], \end{aligned} \quad (7.3)$$

where $\mathcal{N}$ is a normalization constant. The value of $\widetilde{J}(k=0)$ gives the total number of the D0-branes, so we determine $\mathcal{N}$ as $\mathcal{N} = N/4\pi a^2$, then

$$\frac{\widetilde{J}_0(\vec{k})}{\mathcal{T}_{\rm D0}} = N\left[1 - \frac{1}{3!}k^2a^2 + \frac{1}{5!}k^4a^4 + \mathcal{O}(k^6)\right]. \quad (7.4)$$

On the other hand, we substitute the fuzzy S^2 configuration (4.1) into the corrected D0-brane density (7.2), we obtain (for even N)

$$\frac{\widetilde{J}_0(\vec{k})}{N\mathcal{T}_{\rm D0}} = 1 - \frac{N^2-1}{6N^2}k^2R^2 + \frac{N^2-1}{N^4}\left(\frac{3N^2-7}{360} - 32\left(c + \frac{\widetilde{c}}{12}\right)\right)k^4R^4 + \mathcal{O}(k^6). \quad (7.5)$$

Note that there is no correction of order k^2 since no commutator correction can be found in a way consistent with the dimensionality. (A possible dimensionful parameter α' should

[11] Even if this puzzle is resolved, there remains another puzzle: if one starts with a single D0-brane, it seems impossible to generate extended objects, while in the dual D2-brane picture, one can easily consider a spherical D2-D0 bound state with only a single D0-brane charge in total. To show the finite N duality, one has to solve this another puzzle special for the $N=1$ case. Through the embedding of the fuzzy sphere into the fuzzy funnel, this puzzle is related to the fact that the Nahm data becomes trivial for a monopole with a single magnetic charge.

[12] See [45] for a related discussion on correction terms.

not appear here since one can bring R to be very large so that those stringy corrections are negligible.) First let us see that the coefficient of $\mathcal{O}(k^2)$ determines the radius of the single shell uniquely. Comparing (7.4) and (7.5), we find

$$a = R\sqrt{1-\frac{1}{N^2}}\,. \tag{7.6}$$

This is the relation between the physical radius a of the sphere on which the D0-branes are distributed and the normalization R of the fuzzy sphere solution (4.1). Interestingly, using this relation, we find that the spherical locus relation (4.3) is consistent with its physical interpretation without any finite N correction,

$$\sum_{i=1}^{3} X_i^2 = a^2 \mathbb{1}_N\,. \tag{7.7}$$

We compare $\mathcal{O}(k^4)$ term in (7.4) and (7.5). We find that they agree if

$$c + \frac{\widetilde{c}}{12} = \frac{-1}{2880}\,. \tag{7.8}$$

Next, we perform a similar calculation for the fuzzy S^4. Assuming that the D0-brane charge is generated on a single shell of a four-sphere, in the same manner we obtain from the Fourier transformation of a delta function with a support at a radius in the five dimensions $r = \widetilde{a}$,

$$\frac{\widetilde{J}_0(\vec{k})}{\mathcal{T}_{\rm D0}} = N\left[1 - \frac{1}{10}k^2\widetilde{a}^2 + \frac{1}{280}k^4\widetilde{a}^4 + \mathcal{O}(k^6)\right]. \tag{7.9}$$

Here we have already fixed the overall normalization so that the total number of the D0-branes is N. On the other hand, the corrected formula (7.2) gives with the substitution of the fuzzy S^4 configuration (5.1)

$$\frac{\widetilde{J}_0(\vec{k})}{N\mathcal{T}_{\rm D0}} = 1 - \frac{4+n}{10n}k^2R^2 + \frac{n+4}{n^3}\left(\frac{3n^2+12n-8}{840} - 16\left(c+\frac{\widetilde{c}}{5}\right)\right)k^4R^4 + \mathcal{O}(k^6)\,. \tag{7.10}$$

To derive this result, we used the rotational invariance of the expression and also various formulas given in [35]. The comparison of $\mathcal{O}(k^2)$ terms in (7.9) and (7.10) identifies R in terms of $\widetilde{a}$ as

$$\widetilde{a} = R\sqrt{1+\frac{4}{n}}\,. \tag{7.11}$$

This again shows that the correction factor in the four-spherical locus relation (5.5) is due to the wrong normalization of the matrices X_i,

$$\sum_{i=1}^{5} X_i^2 = \widetilde{a}^2 \mathbb{1}_N\,. \tag{7.12}$$

Together with the relation between R and $\widetilde{a}$, the comparison of (7.9) and (7.10) at $\mathcal{O}(k^4)$ provides

$$c + \frac{\widetilde{c}}{5} = \frac{-1}{1680}\,. \tag{7.13}$$

Hence the assumption that the D0-branes should be distributed on a single shell for both the fuzzy S^2 and the fuzzy S^4 can determine the numerical coefficients of the corrections as

$$c = \frac{-1}{5880}\,, \qquad \widetilde{c} = \frac{-5}{2352}\,. \tag{7.14}$$

These are the first nontrivial corrections to the D0-brane charge density formula. Note that these c and $\widetilde{c}$ do not depend on N, as they should be since the charge density formulas themselves are expected to be independent of the dimension of the matrices X_i. In the same manner, corrections to the D2-brane and D4-brane charge density formulas may be obtained.

It is interesting to study the meaning of these correction terms in more detail, and obtain higher order terms such as $\mathcal{O}(k^6)$. Unfortunately, our assumption that the fuzzy S^2 configuration (4.1) (and the fuzzy S^4 configuration (5.1)) may give a single shell in J_0 cannot completely determine the higher order structure, since there appears many correction terms due to the rapid growing of the number of the ways of contracting the indices when a larger number of commutators is considered. Therefore to fix all the coefficients we need some other severe physical principle. It would be intriguing to find it and further investigate the duality between the dual pictures with different worldvolume dimensions.

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A. Democratic duality from tachyon condensation

To find a manifestly democratic description of various fuzzy objects is a different issue beyond the aim of this paper, but in this appendix let us present an example. For the confirmed duality in the flat D2-D0 bound state studied in subsection 2.2, we may find another interesting description which is manifestly democratic in a sense. We use non-BPS D1-branes as a starting point and can make a D2-D0 solution which exhibits the duality.

On parallel N non-BPS D1-branes, we have a real tachyon and transverse scalar fields in adjoint representation of $\mathrm{U}(N)$. In the large-N limit one can construct a flat BPS D2-brane by turning on these fields as [46]

$$T = u\hat{p}_2\,, \qquad \Phi_2 = \hat{x}_2\,, \tag{A.1}$$

where u is a constant parameter which is taken to be infinite to satisfy the equations of motion of a boundary string field theory (BSFT) for the D1-branes, and $\hat{x}_2$ and $\hat{p}_2$ are matrix representation of the Heisenberg algebra

$$[\hat{x}_2, \hat{p}_2] = i\,. \tag{A.2}$$

The first description (i) in section 1 is a D2-brane with a constant magnetic field. In order to turn on the gauge field on the D2-brane solution (A.1), we introduce a fluctuation of the tachyon solution as [47]

$$T = u\big(\hat{p}_2 + A_2(\hat{x}_2, x_1, x_0)\big)\,. \tag{A.3}$$

This is basically a covariant derivative. For our concern, we put $A_2 = f x_1$ to turn on the constant field strength $F_{12} = f$. The resultant configuration

$$T = u\big(\hat{p}_2 + f x_1\big)\,, \qquad \Phi_2 = \hat{x}_2\,, \tag{A.4}$$

is again a solution of the BSFT. This field strength generates D0-branes bound in the D2-brane and their density is determined by f as explained in subsection 2.2.

We notice here that this (A.4) has another interpretation which becomes manifest when we rewrite it as

$$T = fu\big(x_1 - \Phi_1\big)\,, \qquad \Phi_2 = \hat{x}_2\,, \tag{A.5}$$

where $\Phi_1 = -\hat{p}_2/f$. The first equation in (A.5) means a collection of D0-branes since x_1 is a kink on the non-BPS D1-branes, and the location of those D0-branes are specified by eigenvalues of Φ_1 which give zeros of the tachyon field. So, an interpretation of (A.4) is a collection of the D0-branes whose location is given by $x_1 = \Phi_1$ and $x_2 = \Phi_2$. Due to (A.2), we obtain

$$[\Phi_1, \Phi_2] = \frac{i}{f} \tag{A.6}$$

which is what we studied in subsection 2.2. The factor $1/f$ appears consistently. This coincides the second description (ii) in section 1.

Thus we conclude that the solution (A.4) on the non-BPS D1-branes has two meanings, one is a flat D2-brane with the constant field strength (i), and another is a collection of D0-branes satisfying the Heisenberg algebra (ii). The solution (A.4) is a simultaneous manifestation of these two equivalent descriptions.

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